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PROPAGATION OF MULTIWAVELENGTH LASER RADIATION THROUGH ATMOSPHERIC TURBULENCE

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PROPAGATION OF MULTIWAVELENGTH LASER RADIATION THROUGH ATMOSPHERIC TURBULENCE

Oregon Graduate Center

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This report reviews initial progress on a new program of investigation of scintillation and coherence effects for a laser-illuminated, noncooperative target as viewed through atmospheric turbulence. The application of the effort is in the prediction of turbulence effects on the operation of coherent optical adaptive transmitter (COAT) systems.

Significant analytical progress is reviewed, including results for the mutual

coherence function, variance and covariance of irradiance, spectra, and statistics for a coherently illuminated diffuse target. The approach utilizes the extended Huygens-Fresnel principle, and includes turbulence effects in both the path from the transmitter to the target and that back to the receiver. It is found that there are three pertinent covariance scales and six possible parameter realms, and that the normalized variance will be unity except in those cases when the target spot is sufficiently small as to constitute a quasi-point-source. This parameter-realm viewpoint is further explored in relation to a variety of possible sources operating through turbulence. The analysis is partially extended to a more complex target and to the incoherent case, and future analytical tasks and applications to real adaptive systems are outlined.

The establishment of an experimental field facility is also described, which will be capable of measuring all pertinent quantities at both visible and middle-infrared wavelengths. Preliminary experimental results are presented.

The results of this work should aid materially in the understanding of the dynamic behavior of adaptive laser sources operating in the presence of significant atmospheric turbulence.

PROPAGATION OF MULTIWAVELENGTH LASER RADIATION THROUGH ATMOSPHERIC TURBULENCE

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Summary

This report reviews initial progress on a new program of investigation of scintillation and coherence effects for a laser-illuminated , noncooperative target as viewed through atmospheric turbulence. The application of the effort is in the prediction of turbulence effects on the operation of coherent optical adaptive transmitter (COAT) systems.

Significant analytical progress is reviewed, including results for the mutual coherence function, variance and covariance of irradiance, spectra, and statistics for a coherently illuminated diffuse target. The approach utilizes the extended Huygens-Fresnel principle, and includes turbulence effects in both the path from the transmitter to the target and that back to the receiver. It is found that there are three pertinent covariance scales and six possible parameter realms, and that the normalized variance will be unity except in those cases when the target spot is sufficiently small as to constitute a quasi-point-source. This parameter-realm viewpoint is further explored in relation to a variety of possible sources operating through turbulence. The analysis is partially extended to a more complex target and to the incoherent case, and future analytical tasks and applications to real adaptive systems are outlined.

The establishment of an experimental field facility is also described, which will be capable of measuring all pertinent quantities at both visible and middle-infrared wavelengths. Preliminary experimental results are presented.

The results of this work should aid materially in the understanding of the dynamic behavior of adaptive laser sources operating in the presence of significant atmospheric turbulence.

Table of Contents

I. Introduction

This report reviews a new and ongoing investigation of scintillation effects arising from the illumination of noncooperative targets with laser radiation in the presence of atmospheric turbulence. The application of interest is an understanding of the performance of adaptive optical transmitter systems in the presence of target and turbulence-induced speckle and other scintillation structure.

During the initial period of research, a theoretical understanding of turbulence effects was achieved for diffuse targets with coherent and incoherent (highly-multimode) illumination. Progress was also made on the problem of target structure (glints); and on scintillations from generalized sources, including parameter realms and covariance scales. These theoretical results are discussed in detail in Section II.

During the same period, an experimental program was developed for visible and infrared (3.5-3.8 micron) wavelengths. Preliminary field experiments were conducted at 6328Å and a well-instrumented field facility established at 4880Å. Concurrently, a pulsed infrared laser and receiver system was designed and fabrication initiated. The experimental efforts are discussed in Section III.

II. Theoretical Description

Significant progress has been made in the physical and analytical understanding of turbulence effects on the dynamics of target-reflected radiation. The important quantities, relating ultimately to the performance of an adaptive transmitter system, are the variance of irradiance (σ_I^2), covariance of irradiance [$C_I(p)$], mutual coherence function [$\Gamma(p)$], probability distribution function of irradiance, and spatial and temporal power spectrum. These quantities are of interest in the plane of the active laser transceiver, and include turbulence effects on both the illuminating radiation from the transmitter to the target and on the scattered radiation over the return path (Fig. 1).

The treatment to be given below utilizes primarily the extended Huygens-

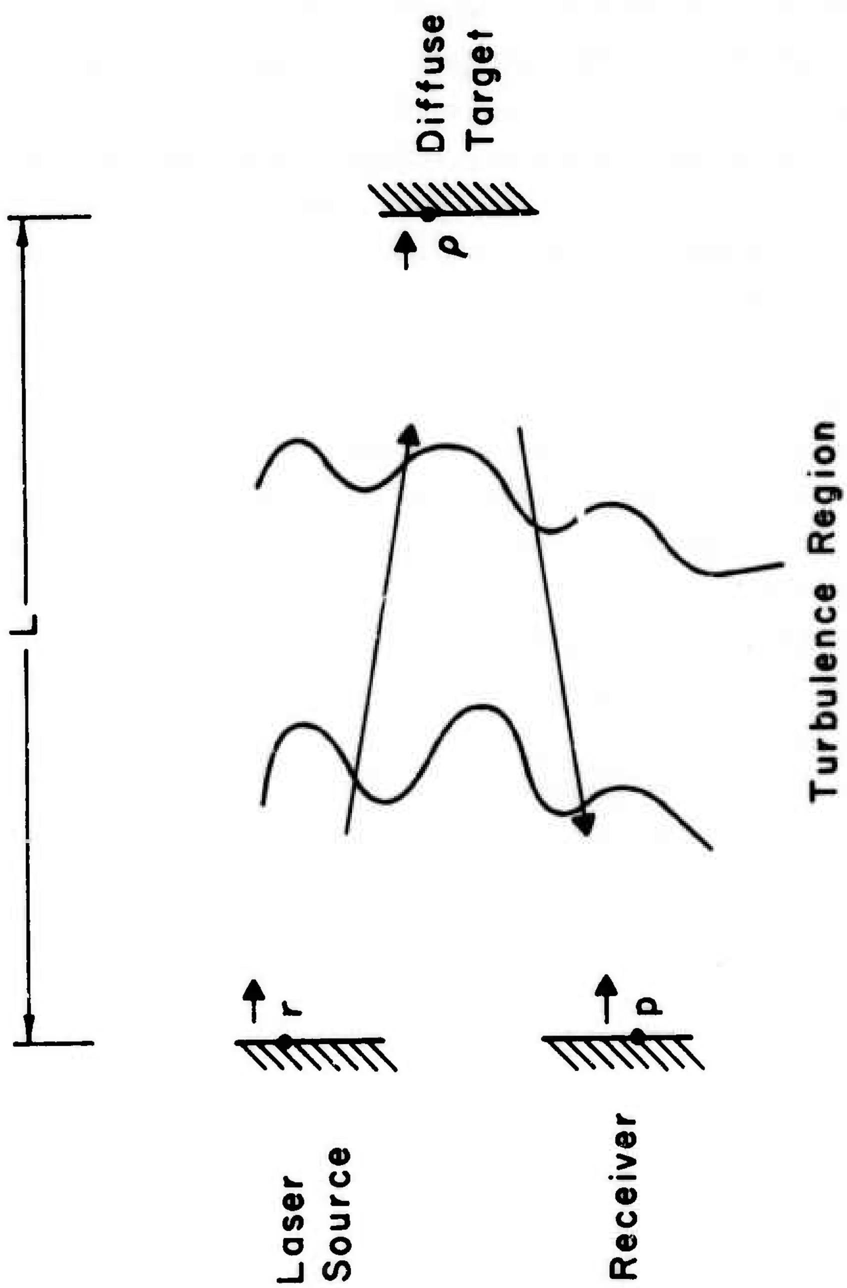

Figure 1. Illuminator, Target, and Receiver Configuration

Fresnel principle as applied to a turbulent path.[1,2] Until otherwise stated, the laser is assumed to be a coherent (TEM_{00}) source, collimated or focused, with a perfectly diffuse target. In all cases, we attempt to show clearly the assumptions and approximations that are made and to discuss their implications. An important feature is the definition of distinct (asymptotic) parameter conditions applying to any given configuration; there are generally six such conditions, representing the possible permutations of inequalities between the three pertinent parameters: Fresnel zone size $(L/k)^{1/2}$, coherence radius (ρ_0) and speckle size in the absence of turbulence. Each such condition will in general carry a distinct physical and analytical interpretation. It will be seen that cases of strong scintillations ("saturation" or multiple scattering) are included in these conditions, so that the treatment is general.

We first consider (Sec. II-A) the most common situation, i.e., that in which the primary effect of the turbulence on the reflected radiation arises through the perturbation of the phase term in the associated Green's function. The implications of this assumption in terms of the field statistics are also explained. A review of some of the developments of this section appeared in the preceding technical report[3] but without a full interpretation of the assumptions.

We then generalize the development (Sec. II-B) to include the effects of the amplitude perturbation term in the Green's function. The implications are again explored in detail. In Sec. II-C, we treat the first-order statistics of irradiance for a target containing one or more glints. Incoherent illumination is then discussed in Sec. II-D.

In Sec. II-E, we relate the current development to an alternative, earlier treatment of a related problem and show that the two approaches are basically complementary. We also consider the possible parameter realms for a variety of source types, with attendant predictions of strengths and spatial scales of scintillations.

Finally, in II-F, we outline further topics and extensions of interest and related efforts to be undertaken.

1. R.F.Lutomirski and H.T.Yura,"Propagation of a Finite Optical Beam in an Inhomogeneous Medium", Applied Optics,10, 1652, July 1971.
2. H.Yura,"Mutual Coherence Function of a Finite Cross Section Optical Beam Propagating in a Turbulent Medium",Applied Optics,11,1399, June 1972.
3. J.R.Kerr, et al, "Propagation of Multiwavelength Laser Radiation through Atmospheric Turbulence", RADC Technical Report, August 1975.

II.A. Basic Irradiance Statistics and Mutual Coherence Function

Previous work on speckle statistics has primarily been concentrated on the nature and statistics of the target surface, propagation of the speckle field without turbulence, and effects of speckle on image quality. Speckle propagation through turbulence has been considered over a vertical path for the purposes of speckle interferometry.[4]

In the present section an analysis is given of the first and second order statistics of the received intensity (irradiance) after scattering from a diffuse target. The treatment is based on the extended Huygens-Fresnel formulation and includes the effects of the turbulent atmosphere on the laser beam as it propagates to the target and on the speckle as it propagates back to the receiver. Formulations are given for both the focused and collimated cases. The analysis also includes the mutual coherence function (MCF).

The source, target, and receiver configuration is shown in Figure 1. The present analysis is confined to the case of a TEM_{00} laser illuminator. The source and target are assumed to be much smaller than the path length (L), and the distance between the receiver and source is greater than the source size and much smaller than the path length. These geometric conditions confine the problem to small angles and ensure that the outgoing and returning radiation experience independent turbulence regions; the latter limitation is thought to be inessential owing to the diffuse target characteristics.

1. Mean Irradiance at Receiver

To find the mean irradiance, we need no assumptions other than that of a diffuse target.

We write the source amplitude distribution as

$$U_o(\bar{r}) = U_o \exp\left(-\frac{r^2}{2\alpha_o^2} - \frac{ikr^2}{2F}\right) \tag{1}$$

where α_o and F are the characteristic beam radius and focal length respec-

4. M. Elbaum, et al, "Laser Correlography: Transmission of High-Resolution Object Signatures through the Turbulent Atmosphere", Riverside Research Institute, Technical Report T-1/306-3-11, October 31, 1974.

tively. The field at the target is written from the extended Huygens-Fresnel principle[1,2] as

$$U(\bar{\rho}) = \frac{ke^{ikL}}{2\pi iL} \int U_o(\bar{r}) \exp\left[\frac{ik|\bar{\rho} - \bar{r}|^2}{2L} + \psi_1(\bar{\rho},\bar{r})\right] d\bar{r} \tag{2}$$

where ψ_1 describes the effects of the random medium on the propagation of a spherical wave. Combining Eqs. (1) and (2), we have

$$U(\bar{\rho}) = \frac{ke^{ik\left[L + \frac{\rho^2}{2L}\right]} U_o}{2\pi iL} \int \exp\left[-\frac{r^2}{2\alpha_o^2} + \frac{ik}{2L}\left(1 - \frac{L}{F}\right) r^2 - \frac{ik}{L} \bar{\rho}\cdot\bar{r}\right.$$

$$\left. + \psi_1(\bar{\rho},\bar{r})\right] d\bar{r} \tag{3}$$

In particular, this applies to the special cases of a focused ($L = F$) or collimated ($F \to \infty$) beam respectively.

The field at the receiver is written by reapplying the Huygens-Fresnel principle to the field at the target:

$$U(\bar{p}) = \frac{ke^{ik\left[L + \frac{p^2}{2L}\right]}}{2\pi iL} \int U'(\bar{\rho}) \exp\left[\frac{ik}{2L}(\rho^2 - 2\bar{p}\cdot\bar{\rho}) + \psi_2(\bar{p},\bar{\rho})\right] d\bar{\rho} \tag{4}$$

where $U'(\bar{\rho})$ is the field solution after reflection from the target, and ψ_2 represents the turbulence effect from the target to the receiver. The mean intensity at the receiver is then

$$\langle I(\bar{p})\rangle = \langle|U(\bar{p})|^2\rangle = \left(\frac{k}{2\pi L}\right)^2 \iint d\bar{\rho}_1 d\bar{\rho}_2 \langle U'(\bar{\rho}_1) U'^*(\bar{\rho}_2)\rangle$$

$$\cdot \exp\left[\frac{ik}{2L}\left((\rho_1^2 - \rho_2^2) - 2\bar{p}\cdot(\bar{\rho}_1 - \bar{\rho}_2)\right)\right] \tag{5}$$

$$\cdot \left\langle \exp\left[\psi_2(\bar{p},\bar{\rho}_1) + \psi_2^*(\bar{p},\bar{\rho}_2)\right]\right\rangle$$

Through the assumption of a diffuse target, the reflected beam suffers a random phase delay from point-to-point over the target, so that

$$<U'(\bar{\rho}_1)U'^*(\bar{\rho}_2)> = <I(\bar{\rho}_1)> \delta(\bar{\rho}_1-\bar{\rho}_2) \tag{6}$$

Using this in Eq. (5), the mean intensity becomes

$$<I(\bar{p})> = \left(\frac{k}{2\pi L}\right)^2 \int d\bar{\rho}_1 <|U(\bar{\rho}_1)|^2> < \exp\left[\psi_2(\bar{p},\bar{\rho}_1) + \psi_2^*(\bar{p},\bar{\rho}_1)\right]> \tag{7}$$

where the mean exponential term is unity from considerations of energy conservation.[2] The resultant mean intensity at the receiver is then simply

$$<I(\bar{p})> = \left(\frac{k}{2\pi L}\right)^2 \int d\bar{\rho} <|U(\bar{\rho})|^2> \tag{8}$$

To complete the solution, we use Eq. (3) with Eq. (8). We note that the structure function gives us ($r = |\bar{r}_1-\bar{r}_2|$)

$$< \exp\left[\psi_1(\bar{\rho},\bar{r}_1) + \psi_1^*(\bar{\rho},\bar{r}_2)\right]> = e^{-\left(\frac{r}{\rho_o}\right)^{5/3}} \tag{9}$$

For the focused beam, we then have

$$<|U(\bar{\rho})|^2> = \left(\frac{k}{2\pi L}\right)^2 U_o^{\,2} \iint d\bar{r}_1 d\bar{r}_2 \exp\left[-\frac{r_1^2 + r_2^2}{2\alpha_o^{\,2}} - \frac{ik}{L}\bar{\rho}\cdot(\bar{r}_1-\bar{r}_2) - \left(\frac{r}{\rho_o}\right)^{5/3}\right] \tag{10}$$

Carrying out the integration indicated in Eq. (8), involving the Fourier-Bessel integral, we have finally

$$<I(\bar{p})> = \frac{1}{2\pi} \left(\frac{k}{L}\right)^2 U_o^{\,2} \frac{\alpha_o^2}{2} \tag{11}$$

The result for the collimated beam is identical, and in fact could be deduced for an arbitrary beam focus (Eq. (1)) through conservation of energy:

$$\langle I(\bar{p})\rangle = \left(\frac{k}{2\pi L}\right)^2 \int d\bar{r} \, \langle |U(\bar{r})|^2 \rangle$$

$$= \left(\frac{k}{2\pi L}\right)^2 2\pi U_o^2 \int_o^\infty r e^{-r^2/\alpha_o^2} dr = \frac{1}{2\pi}\left(\frac{1}{L}\right)^2 U_o^2 \frac{\alpha_o^2}{2} \tag{12}$$

Thus the mean intensity at the receiver (illuminator) plane is uniform and independent of turbulence level.

2. Correlation Function of Irradiance

In order to calculate the correlation function or covariance of irradiance, we assume for the present section that the perturbation Green's function (wave structure function) is dominated by the phase perturbation (phase structure function).[5] This will be true for many cases of interest and will be relaxed in subsequent sections, where the actual implications of the assumption are pointed out.

The correlation function of the intensity at receiver points $\bar{p}_1$ and $\bar{p}_2$ is given by

$$B_I(\bar{p}_1,\bar{p}_2) = \langle I_1(\bar{p}_1) I_2(\bar{p}_2)\rangle = \langle U(\bar{p}_1) U^*(\bar{p}_1) U(\bar{p}_2) U^*(\bar{p}_2)\rangle \tag{13}$$

Utilizing the extended Huygens-Fresnel principle, the correlation function can be expressed as

$$B_I(\bar{p}_1,\bar{p}_2) = \left(\frac{k}{2\pi L}\right)^4 \iiiint d\bar{\rho}_1 d\bar{\rho}_2 d\bar{\rho}_3 d\bar{\rho}_4 \, \langle U(\bar{\rho}_1) U^*(\bar{\rho}_2) U(\bar{\rho}_3) U^*(\bar{\rho}_4)\rangle$$

$$\exp[ik(R_1 - R_2 + R_3 - R_4)] \, H(\bar{\rho}_1,\bar{\rho}_2,\bar{\rho}_3,\bar{\rho}_4;\bar{p}_1,\bar{p}_2) \tag{14}$$

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5. R. S. Lawrence and J. W. Strohbehn, "A Survey of Clean-Air Propagation Effects Relevant to Optical Communications", Proc. IEEE, 58, 1523, October 1970.

where H is the fourth order mutual coherence function given by

$$H = \langle \exp[\psi(\bar{\rho}_1,\bar{p}_1) + \psi^*(\bar{\rho}_2,\bar{p}_1) + \psi(\bar{\rho}_3,\bar{p}_2) + \psi^*(\bar{\rho}_4,\bar{p}_2)] \rangle \tag{15}$$

and

$$R_1 = |\bar{p}_1 - \bar{\rho}_1|$$
$$R_2 = |\bar{p}_1 - \bar{\rho}_2|$$
$$R_3 = |\bar{p}_2 - \bar{\rho}_3|$$
$$R_4 = |\bar{p}_2 - \bar{\rho}_4|$$

Under the assumption of dominant phase perturbations,

$$H = \langle \exp[i\phi(\bar{\rho}_1,\bar{p}_1) - i\phi(\bar{\rho}_2,\bar{p}_1) + i\phi(\bar{\rho}_3,\bar{p}_2) - i\phi(\bar{\rho}_4,\bar{p}_2)] \rangle \tag{16}$$

After reflection from the diffuse target, the fields are Gaussian and spatially incoherent. Therefore, the fields at the target can be expressed as

$$\begin{aligned}\langle U(\bar{\rho}_1)U^*(\bar{\rho}_2)U(\bar{\rho}_3)U^*(\bar{\rho}_4)\rangle &= \langle U(\bar{\rho}_1)U^*(\bar{\rho}_2)\rangle\ \langle U(\bar{\rho}_3)U^*(\bar{\rho}_4)\rangle \\ &+ \langle U(\bar{\rho}_1)U^*(\bar{\rho}_4)\rangle\ \langle U^*(\bar{\rho}_2)U(\bar{\rho}_3)\rangle \\ &= \langle I(\bar{\rho}_1)\rangle\ \langle I(\bar{\rho}_3)\rangle\ \delta(\bar{\rho}_1-\bar{\rho}_2)\ \delta(\bar{\rho}_3-\bar{\rho}_4) \\ &+ \langle I(\bar{\rho}_1)\rangle\ \langle I(\bar{\rho}_3)\rangle\ \delta(\bar{\rho}_1-\bar{\rho}_4)\ \delta(\bar{\rho}_3-\bar{\rho}_2)\end{aligned} \tag{17}$$

Utilizing (17) and (14) the correlation function can be expressed as

$$\begin{aligned}B_I(\bar{p}_1,\bar{p}_2) &= \left(\frac{k}{2\pi L}\right)^4 \iint d\bar{\rho}_2\, d\bar{\rho}_4\, \langle I(\bar{\rho}_2)\rangle\ \langle I(\bar{\rho}_4)\rangle \\ &+ \left(\frac{k}{2\pi L}\right)^4 \iint d\bar{\rho}_2\, d\bar{\rho}_4\, \langle I(\bar{\rho}_2)\rangle\ \langle I(\bar{\rho}_4)\rangle \exp\left[\frac{ik}{L}(\bar{\rho}_2-\bar{\rho}_4)\cdot(\bar{p}_1-\bar{p}_2)\right] \\ &\cdot H(\bar{\rho}_2,\bar{\rho}_2,\bar{\rho}_4,\bar{\rho}_4;\bar{p}_1,\bar{p}_2)\end{aligned} \tag{18}$$

where use has been made of

$$R_1-R_2 \simeq \frac{1}{2L}\left[\rho_1^2 - \rho_2^2 - 2(\bar{\rho}_1-\bar{\rho}_2)\cdot\bar{p}_1\right]$$

$$R_3-R_4 \simeq \frac{1}{2L}\left[\rho_3^2 - \rho_4^3 - 2(\bar{\rho}_3 - \bar{\rho}_4)\cdot\bar{p}_1\right]$$

The fourth order mutual coherence function (see Appendix A) in Eq. (18) is given by[2,6,7]

$$H(\bar{\rho}_2,\bar{\rho}_2,\bar{\rho}_4,\bar{\rho}_4;\bar{p}_1,\bar{p}_2) = H(\bar{\rho}_1,\bar{\rho}_2,\bar{\rho}_3,\bar{\rho}_4;\bar{p}_1,\bar{p}_2,\bar{p}_3,\bar{p}_4)\Bigg|_{\substack{\bar{\rho}_1 = \bar{\rho}_2 \\ \bar{\rho}_3 = \bar{\rho}_4 \\ \bar{p}_3 = \bar{p}_1 \\ \bar{p}_4 = \bar{p}_2}}$$

$$= \left. e^{\frac{1}{2}(D_{12} - D_{13} + D_{14} + D_{23} - D_{24} + D_{34})}\right|_{\substack{\bar{\rho}_1 = \bar{\rho}_2 \\ \bar{\rho}_3 = \bar{\rho}_4 \\ \bar{p}_3 = \bar{p}_1 \\ \bar{p}_4 = \bar{p}_2}} \qquad (19)$$

where the wave phase structure functions D_{ij} are given by

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6. D. L. Fried, "Effects of Atmospheric Turbulence on Static and Tracking Optical Heterodyne Receivers", Optical Science Consultants, Technical Report TR-027, August 1971.
7. D. L. Fried, "Atmospheric Modulation Noise in an Optical Heterodyne Receiver", IEEE Trans. on Quantum Electronics, QE-3, 213, June 1967.

$$D_{ij} = \frac{1}{\rho_o{}^{5/3}} \left[\frac{\left| |\bar{p}_j - \bar{p}_i|^{8/3} - |\bar{\rho}_j - \bar{\rho}_i|^{8/3} \right|}{|(\bar{p}_j - \bar{p}_i) - (\bar{\rho}_j - \bar{\rho}_i)|} \right] \tag{20}$$

where

$\rho_o = (0.545\ C_n{}^2 L k^2)^{-3/5}$ is the turbulence-induced coherence scale and

$C_n{}^2$ = Structure constant of index of refraction ($m^{-2/3}$).

Using this in (18) and making the change of variables

$$\bar{\rho} = \bar{\rho}_2 - \bar{\rho}_4 \ , \ p = \bar{p}_1 - \bar{p}_2$$

and

$$2\bar{R} = \bar{\rho}_2 + \bar{\rho}_4$$

and recognizing that the first term in (18) equals $<I(\bar{p}_1)><I(\bar{p}_2)>$, the covariance for the focused case is given by

$$C_I(\bar{p}_1,\bar{p}_2) = \left(\frac{k}{2\pi L}\right)^4 \left(\frac{k}{L}\right)^4 |U_o|^4 \left(\frac{\alpha_o{}^2}{2}\right)^2 e^{-2\left(\frac{p}{\rho_c}\right)^{5/3}}$$

$$\cdot \iint d\bar{\rho}\ d\bar{R}\ J_o\left(\frac{k}{L} r_1 \left|\bar{R} + \frac{\bar{\rho}}{2}\right|\right) J_o\left(\frac{k}{L} r_2 \left|\bar{R} - \frac{\bar{\rho}}{2}\right|\right) e^{\frac{ik}{L}\bar{\rho}\cdot\bar{p}}$$

$$\cdot \iint r_1 r_2 dr_1 dr_2 \exp\left[-\frac{(r_1{}^2 + r_2{}^2)}{4\alpha_o{}^2} - \frac{(r_1{}^{5/3} + r_2{}^{5/3})}{\rho_o{}^{5/3}} \right.$$

$$\left. - \frac{2}{\rho_o{}^{5/3}} \left[\rho^{5/3} - \frac{1}{2} \frac{|p^{8/3} - \rho^{8/3}|}{|\bar{p} - \bar{\rho}|} - \frac{1}{2} \frac{|p^{8/3} - \rho^{8/3}|}{|\bar{p} + \bar{\rho}|} \right] \right] \tag{21}$$

The covariance for the collimated case is obtained from (21) by replacing the $dr_1 dr_2$ integration by

$$\iint r_1 r_2 dr_1 dr_2 \exp\left[-(r_1^2 + r_2^2)\left(\left(\frac{1}{2\alpha_o}\right)^2 + \left(\frac{k\alpha_o}{2L}\right)^2\right) - \frac{r_1^{5/3} + r_2^{5/3}}{\rho_o^{5/3}}\right.$$

$$\left. - \frac{2}{\rho_o^{5/3}}\left[\rho^{5/3} - \frac{1}{2}\frac{|p^{8/3} - \rho^{8/3}|}{|\bar{p} - \bar{\rho}|} - \frac{1}{2}\frac{|p^{8/3} - \rho^{8/3}|}{|\bar{p} + \bar{\rho}|}\right]\right] \tag{22}$$

In order to further reduce the number of integrations, the zero order Bessel functions must be expanded to functionally separate the $\bar{R}$ and $\bar{\rho}$ dependence. This can be accomplished by utilizing the following identities:

$$J_o\left(\frac{k}{L} r_1 \left|\bar{R} \pm \frac{\bar{\rho}}{2}\right|\right) = \sum_{m=0}^{\infty} \epsilon_m (\mp 1)^m J_m\left(\frac{k}{L} r_1 R\right) J_m\left(\frac{k}{L} r_1 \frac{\rho}{2}\right) \cos m\phi$$

where

$$\phi = \theta_R - \theta_\rho$$

$$\epsilon_o = 1$$

and

$$\epsilon_m = 2 \qquad \text{for } m \neq 0. \tag{23}$$

The θ_R integration is then given by

$$\int_0^{2\pi} J_o\left(\frac{k}{L} r_1 \left|\bar{R} + \frac{\bar{\rho}}{2}\right|\right) J_o\left(\frac{k}{L} r_2 \left|\bar{R} - \frac{\bar{\rho}}{2}\right|\right) d\theta_R$$

$$= 2\pi \sum_{m=0}^{\infty} (-1)^m \epsilon_m J_m\left(\frac{k}{L} r_1 R\right) J_m\left(\frac{k}{L} r_2 R\right)$$

$$\cdot J_m\left(\frac{k}{L} r_1 \frac{\rho}{2}\right) J_m\left(\frac{k}{L} r_2 \frac{\rho}{2}\right) \tag{24}$$

Using the Fourier-Bessel integral,

$$\int_0^{\infty} r_1 dr_1 f(r_1) \int_0^{\infty} RdR\, J_m\left(\frac{k}{L} r_1 R\right) J_m\left(\frac{k}{L} r_2 R\right) = \left(\frac{L}{k}\right)^2 f(r_2) \text{ for } r_2 \neq 0$$

$$= \frac{1}{2}\left(\frac{L}{k}\right)^2 f(o) \text{ for } r_2 = 0. \tag{25}$$

and the covariance for the focused case becomes

$$C_{I\,\text{focused}}(\bar{p}) = \left(\frac{1}{2\pi}\right)^3 \left(\frac{k}{L}\right)^6 |U_o|^4 \left(\frac{\alpha_o}{2}\right)^2 \exp\left[-2\left(\frac{p}{\rho_o}\right)^{5/3}\right] \int_o^{\infty} r_2 dr_2$$

$$\cdot \exp\left[-\frac{r_2^{\,2}}{2\alpha_o^{\,2}} - \frac{2r_2^{\,5/3}}{\rho_o^{\,5/3}}\right] \int d\bar{\rho}\, J_o\left(\frac{k}{L} r_2 \rho\right) \left[1 - \frac{1}{2}\delta_{r_2}\right]$$

$$\cdot \exp\left[\frac{ik}{L}\bar{p}\cdot\bar{\rho} - \frac{2}{\rho_o^{\,5/3}}\left[\rho^{5/3} - \frac{1}{2}\frac{\left|p^{8/3} - \rho^{8/3}\right|}{|\bar{p} - \bar{\rho}|} - \frac{1}{2}\frac{\left|p^{8/3} - \rho^{8/3}\right|}{|\bar{p} + \bar{\rho}|}\right]\right] \tag{26}$$

where use has been made of

$$\sum_{m=0}^{\infty} (-1)^m \epsilon_m J_m^{\,2}(x) = J_o(2x) \tag{27}$$

and δ_{r_2} is the Kronecker delta. The variance is then given by

$$\sigma_I^2 = \left(\frac{1}{2\pi}\right)^2 \left(\frac{k}{L}\right)^4 |U_o|^4 \left(\frac{\alpha_o^2}{2}\right)^2 \tag{28}$$

and the normalized variance

$$\sigma_{I_N}^2 = \frac{\sigma_I^2}{<I>^2}$$

is unity (Eq. 12).

The same technique yields the covariance for the collimated case. It is given by

$$\underset{\text{Collimated}}{C_I(\bar{p})} = \left(\frac{1}{2\pi}\right)^3 \left(\frac{k}{L}\right)^6 |U_o|^4 \left(\frac{\alpha_o^2}{2}\right)^2 \exp\left[-2\left(\frac{p}{\rho_o}\right)^{5/3}\right] \int_0^\infty r_2 dr_2$$

$$\cdot \exp\left[-2r_2^2\left[\left(\frac{1}{2\alpha_o}\right)^2 + \left(\frac{k\alpha_o}{2L}\right)^2\right] - \frac{2r_2^{5/3}}{\rho_o^{5/3}}\right] \int d\bar{\rho}\, J_o\left(\frac{k}{L} r_2 \rho\right)$$

$$\cdot \left[1 - \frac{1}{2}\delta_{r_2}\right]$$

$$\cdot \exp\left[\frac{ik}{L}\bar{p}\cdot\bar{\rho} - \frac{2}{\rho_o^{5/3}}\left[\rho^{5/3} - \frac{1}{2}\frac{|p^{8/3} - \rho^{8/3}|}{|\bar{p} - \bar{\rho}|} - \frac{1}{2}\frac{|p^{8/3} - \rho^{8/3}|}{|\bar{p} + \bar{\rho}|}\right]\right] \tag{29}$$

and the normalized variance is again unity.

The above results are based on the assumptions of a diffuse target and phase perturbations dominating the turbulence effects. The resulting threefold integrations will be numerically evaluated in future work. Physical interpretation is not difficult but will be clarified in the discussion of further simplifications below. It suffices to point out here that the covariance fundamentally involves two scales: the turbulence-induced coherence scale (ρ_o) and the speckle scale in the absence of turbulence (α_o and $L/k\alpha_o$ for the focused and collimated case respectively).

Also, we note that within the present assumptions, the normalized variance of intensity is <u>always unity</u> independent of turbulence strength. This agrees with a physical model of identical-frequency, randomly phased oscillators summed to represent any given point in the receiver field: the model applies regardless of whether target speckle (α_o or $L/k\alpha_o$) or "atmospheric speckle" (ρ_o) dominates.

3. Mutual Coherence Function

The mutual coherence function (MCF) may be very important in analyzing the operation of a coherent optical adaptive system, and can be readily derived given the assumption of a diffuse target but without assuming dominance of the phase perturbation term. We write

$$\Gamma(\bar{p}_1,\bar{p}_2) = \left(\frac{k}{2\pi L}\right)^2 \iint d\bar{\rho}_1 d\bar{\rho}_2 < U(\bar{\rho}_1)U^*(\bar{\rho}_2) > \exp\left\{ik\left[R_1(\bar{\rho}_1,\bar{p}_1) - R_2(\bar{\rho}_2,\bar{p}_2)\right]\right\} < \exp\left[\psi_2(\bar{p}_1,\bar{\rho}_1) + \psi_2^*(\bar{p}_2,\bar{\rho}_2)\right] > \qquad (30)$$

where $R_1(\bar{\rho}_1,\bar{p}_1)$, $R_2(\bar{\rho}_2,\bar{p}_2)$ are the distances from $\bar{\rho}_1$ to $\bar{p}_1$ and $\bar{\rho}_2$ to $\bar{p}_2$ respectively.

By the Fresnel approximation

$$R_1(\bar{\rho}_1,\bar{p}_1) - R_2(\bar{\rho}_2,\bar{p}_2) \cong \frac{p_1^2 - p_2^2 + \rho_1^2 - \rho_2^2}{2L} - \frac{\bar{p}_1 \cdot \bar{\rho}_1 - \bar{p}_2 \cdot \bar{\rho}_2}{L} \qquad (31)$$

Finally, from (30) and (31),

$$\Gamma(\bar{p}_1,\bar{p}_2) = \left(\frac{k}{2\pi L}\right)^2 \exp\left[\frac{ik(p_1^2-p_2^2)}{2L}\right]\iint d\bar{\rho}_1 d\bar{\rho}_2 \quad \langle U(\bar{\rho}_1)U^*(\bar{\rho}_2)\rangle$$

$$\cdot \exp\left\{ik\left(\frac{\rho_1^2-\rho_2^2}{2L} - \frac{\bar{p}_1\cdot\bar{\rho}_1-\bar{p}_2\cdot\bar{\rho}_2}{L}\right)\right\} \langle \exp\left[\psi_2(\bar{p}_1,\bar{\rho}_1) + \psi_2{}^*(\bar{p}_2,\bar{\rho}_2)\right]\rangle \tag{32}$$

Since the wave is incoherent after reflection from the diffuse target, the coherence function at that plane can again be represented by the Dirac delta function as given in (6). Using this in (32), $\Gamma(\bar{p}_1,\bar{p}_2)$ can be simplified to

$$\Gamma(\bar{p}_1,\bar{p}_2) = \left(\frac{k}{2\pi L}\right)^2 \exp\left[\frac{ik(p_1^2-p_2^2)}{2L}\right]\int d\bar{\rho} \langle I(\bar{\rho})\rangle \exp\left\{-\frac{ik}{L}(\bar{p}_1-\bar{p}_2)\cdot\bar{\rho}\right\}$$

$$\cdot e^{-\left(\frac{p}{\rho_o}\right)^{5/3}} \tag{33}$$

In the absence of turbulence, this equation is entirely identical to the Van Cittert-Zernike theorem of coherence theory,[9] which is identical to a result obtained by Goodman for the mutual coherence function of a pulsed optical radar.[10]

To complete the solution, we utilize the mean intensity at the target. For the focused case:

$$\langle I(\bar{\rho})\rangle = \left(\frac{k}{L}\right)^2 |U_o|^2 \frac{\alpha_o^2}{2}\int_o^\infty r\,dr\,J_o\left(\frac{k}{L}\rho r\right) e^{-\frac{r^2}{4\alpha_o^2} - \left(\frac{r}{\rho_o}\right)^{5/3}}$$

We thus have

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9. M. Born and E. Wolf, Principles of Optics, Pergamon Press, New York, 1975.
10. J. W. Goodman, "Some Effects of Target-Induced Scintillation on Optical Radar Performance", Proc. IEEE, 53, 1688, 1965.

$$\Gamma(p_1,p_2) = \left(\frac{k}{2\pi L}\right)^2 \left(\frac{k}{L}\right)^2 |U_o|^2 \frac{\alpha_o^2}{2} \int d\bar{\rho} \int_0^\infty r\,dr\, J_o\left(\frac{k}{L}\,\rho r\right)$$

$$\cdot\ e^{-\frac{r^2}{4\alpha_o^2} - \left(\frac{r}{\rho_o}\right)^{5/3}} \exp\left[-\frac{ik}{L}\bar{\rho}\cdot\bar{p} - \left(\frac{p}{\rho_o}\right)^{5/3}\right]$$

$$\cdot\ \exp\left[\frac{ik(p_1^2-p_2^2)}{2L}\right]$$

$$= \frac{1}{2\pi}\left(\frac{k}{L}\right)^4 |U_o|^2 \frac{\alpha_o^2}{2} \int_0^\infty \rho d\rho \int_0^\infty r dr\, J_o\left(\frac{k}{L}\,\rho r\right) J_o\left(\frac{k}{L}\,\rho p\right)$$

$$\cdot\ e^{-\frac{r^2}{4\alpha_o^2} - \left(\frac{r}{\rho_o}\right)^{5/3} - \left(\frac{p}{\rho_o}\right)^{5/3} + \frac{ik}{2L}\left(p_1^2-p_2^2\right)} \qquad (34)$$

From the Fourier-Bessel integral formula,

$$\int_0^\infty \rho J_o\left(\frac{k}{L}\,r\rho\right) J_o\left(\frac{k}{L}\,p\rho\right) d\rho = \left(\frac{L}{k}\right)^2 \frac{1}{\sqrt{rp}}\ \delta(r-p) \qquad (35)$$

Equation (34) can then be simplified and it becomes

$$\Gamma(\bar{p}_1,\bar{p}_2)_{\text{focused}} = \frac{1}{2\pi}\left(\frac{k}{L}\right)^2 |U_o|^2 \frac{\alpha_o^2}{2}\, e^{-\frac{p^2}{4\alpha_o^2} - 2\left(\frac{p}{\rho_o}\right)^{5/3} + \frac{ik}{2L}\left(p_1^2-p_2^2\right)}$$

$$= \langle I(p)\rangle\, e^{-\frac{p^2}{4\alpha_o^2} - 2\left(\frac{p}{\rho_o}\right)^{5/3} + \frac{ik}{2L}\left(p_1^2-p_2^2\right)} \qquad (36)$$

Using $\langle I(\bar{p})\rangle$ for the collimated case in (33) and simplifying yields

$$\Gamma(\bar{p}_1,\bar{p}_2) = \langle I(\bar{p})\rangle \, e^{-p^2\left[\left(\frac{1}{2a_o}\right)^2 + \left(\frac{ka_c}{2L}\right)\right]^2 - 2\left(\frac{p}{\rho_c}\right)^{5/3} + \frac{ik}{2L}\left(p_1^2-p_2^2\right)}$$

Collimated (37)

These results for the MCF will be used further below.

It may be noted that the MCF's of Eq. (36) and (37) imply a "white" or constant spatial power spectrum for the (complex) amplitude. This is of course an idealization resulting from assuming delta-function rather than wavelength-sized phase correlation for the field upon reflection off the target. The more interesting spectrum, however, is that of the irradiance, as discussed below.

4. Probability Distribution

In order to formulate the probability distribution for the scintillating energy at the receiver, we evaluate the n^{th} moment of the intensity in terms of $\langle I(\bar{p})\rangle$. We again assume that the phase perturbations are the dominant turbulence effect, and write the second moment as

$$\langle I^2(\bar{p})\rangle = \left(\frac{k}{2\pi L}\right)^4 \iiiint d\bar{\rho}_1 d\bar{\rho}_2 d\bar{\rho}_3 d\bar{\rho}_4$$

$$\cdot \langle U(\bar{\rho}_1)U^*(\bar{\rho}_2)U(\bar{\rho}_3)U^*(\bar{\rho}_4)\rangle \exp\left[\frac{ik}{2L}\left(\rho_1^2 - \rho_2^2\right.\right.$$

$$\left.\left. + \rho_3^2 - \rho_4^2 - 2(\bar{\rho}_1-\bar{\rho}_2)\cdot\bar{p} - 2(\bar{\rho}_3-\bar{\rho}_4)\cdot\bar{p}\right)\right]$$

$$\cdot \left\langle \exp\left[i\left(\phi(\bar{\rho}_1,\bar{p}) - \phi(\bar{\rho}_2,\bar{p}) + \phi(\bar{\rho}_3,\bar{p}) - \phi(\bar{\rho}_4,\bar{p})\right)\right]\right\rangle \qquad (39)$$

Since the fields after reflection from the diffuse target are jointly Gaussian and uncorrelated,

$$<U(\bar{\rho}_1)U^*(\bar{\rho}_2)U(\bar{\rho}_3)U^*(\bar{\rho}_4)> = <U(\bar{\rho}_1)U^*(\bar{\rho}_2)> <U(\bar{\rho}_3)U^*(\bar{\rho}_4)>$$

$$+ <U(\bar{\rho}_1)U^*(\bar{\rho}_4)> <U^*(\bar{\rho}_2)U(\bar{\rho}_3)>$$

$$= I(\bar{\rho}_1)\delta(\bar{\rho}_1-\bar{\rho}_2)\, I(\bar{\rho}_3)\delta(\bar{\rho}_3-\bar{\rho}_4) + I(\bar{\rho}_1)\delta(\bar{\rho}_1-\bar{\rho}_4)I(\bar{\rho}_3)\delta(\bar{\rho}_3-\bar{\rho}_2) \quad (40)$$

Using (40) in (38) yields

$$<I^2(\bar{p})> = 2\left(\frac{k}{2\pi L}\right)^4 \iint d\bar{\rho}_2 d\bar{\rho}_4 \; <I(\bar{\rho}_2)> <I(\bar{\rho}_4)> \quad (41)$$

Since

$$<I(\bar{p})> = \left(\frac{k}{2\pi L}\right)^2 \int d\bar{\rho}_2 \; <I(\bar{\rho}_2)>$$

the second moment becomes

$$<I^2(\bar{p})> = 2<I(\bar{p})>^2 \quad (42)$$

Similarly, it may be shown that the n^{th} moment is given by

$$<I^n(p)> = N! \; <I(\bar{p})>^N \quad (43)$$

The probability density function for the intensity therefore is exponential and

$$p_I(\alpha) = \frac{e^{-\frac{\alpha}{<I>}}}{<I>} \quad (44)$$

where α is greater than or equal to zero. It is thus concluded that the field and amplitude at the receiver are normally and Rayleigh distributed respectively, given the assumption that phase perturbations dominate the turbulence effects.

5. Simplification for Weak and Strong Turbulence

Since, as shown above, the fields at the receiver are gaussian and spatially "white", it is tempting to assume that the receiver fields are also jointly gaussian. This turns out to be a good approximation in many situations, and in this section the implications and conditions for validity of this added assumption are explored. This leads to a simple, straightforward interpretation of the terms in the covariance of intensity.

The jointly Gaussian assumption yields

$$B_I = \langle U(\bar{p}_1)U^*(\bar{p}_1)\rangle\langle U(\bar{p}_2)U^*(\bar{p}_2)\rangle + \langle U(\bar{p}_1)U^*(\bar{p}_2)\rangle\langle U^*(\bar{p}_1)U(\bar{p}_2)\rangle$$

$$\equiv \langle I(\bar{p}_1)\rangle\langle I(\bar{p}_2)\rangle + |\Gamma(\bar{p}_1,\bar{p}_2)|^2 \tag{45}$$

It follows that the covariance of intensity is given by

$$C_I(\bar{p}_1,\bar{p}_2) = B_I(\bar{p}_1,\bar{p}_2) - \langle I(\bar{p}_1)\rangle\langle I(\bar{p}_2)\rangle = |\Gamma(\bar{p}_1,\bar{p}_2)|^2 \tag{46}$$

Finally, utilizing the mutual coherence result (Eq. 36) the normalized covariance function of irradiance for the focused case can thus be written:

$$\underset{\text{focused}}{C_{I_N}(\bar{p})} \equiv \frac{C_I(\bar{p})}{\sigma_I^{\,2}} = e^{-\frac{p^2}{2\alpha_o^{\,2}} - 4\left(\frac{p}{\rho_o}\right)^{5/3}} \tag{47}$$

where the normalized variance is unity as before.

For the collimated case, the same variance is obtained, and the normalized covariance is

$$\underset{\text{collimated}}{C_{I_N}(\bar{p})} = e^{-4\left(\frac{p}{\rho_o}\right)^{5/3} - \frac{1}{2}\left[\left(\frac{1}{\alpha_o}\right)^2 + \left(\frac{k\alpha_c}{L}\right)^2\right]p^2} \tag{48}$$

The covariance scale lengths are obvious from these results. Either the "atmospheric speckle" (ρ_0) or the "target speckle" (speckle in the absence of turbulence) will predominate, depending upon which is smaller (strong and weak turbulence respectively). We point out in passing that a third covariance scale $(L/k)^{1/2}$ may also enter, but this scale is lost within the present assumption of dominant phase perturbations (see Sections II-B and II-E below).

We note that the spatial power spectrum of irradiance may be readily obtained by transforming Eqs.(47,48). However, a more important quantity in the operation of e.g. an adaptive optical system may be the temporal spectrum. This spectrum, which will be derived in a later section, depends only on the atmospheric speckle term; the target speckle field will not translate with the transverse wind.

We now explore the conditions for validity of the jointly gaussian assumption. The simple multiplicative terms for the covariance scales in Eqs. (47, 48) are replaced by a more complicated interrelationship in Eqs. (26,29). It may therefore be surmised that the jointly gaussian assumption is valid under conditions of weak and strong turbulence, when target and atmospheric speckle terms respectively predominate, but that the jointly gaussian description is not correct in the range of intermediate turbulence effects when both scales are important and interact. We now show that this is indeed the case.

<u>Weak Turbulence</u>

For the weak turbulence case, $\rho_0 \gg \sqrt{L/k}$ and the term

$$J_o\left(\frac{k}{L}\, r\rho\right) e^{-\frac{2}{\rho_o^{5/3}}\left[\rho^{5/3} - \frac{1}{2}\frac{|p^{8/3} - \rho^{8/3}|}{|\bar{p}-\bar{\rho}|} - \frac{1}{2}\frac{|p^{8/3} - \rho^{8/3}|}{|\bar{p}+\bar{\rho}|}\right]} \simeq J_o\left(\frac{k}{L}\, r\rho\right) \tag{49}$$

in (26) and (29). The covariances then become identical to those derived using the jointly gaussian assumption.

Strong Turbulence

In the strong turbulence case, $\rho_o << \sqrt{L/k}$, which corresponds to multiple scattering, or "saturation of scintillations" for a point source. Let us consider Eq. (26) with $\rho_o \to 0$. The only interesting range of the argument (p) is $0 \leq p \lesssim \rho_o$. The Bessel term ($\sim \rho J_o$ in polar coordinates) is appreciable only for $\rho \not\approx 0$, i.e. $\rho >> \rho_o$ or $\rho >> p$; and because of the latter condition the final bracket in the equation is zero. Hence the condition (49) is again obtained, and the covariances again become identical to those derived using the jointly gaussian assumption. The atmosphere has "decoherentized" the field in a manner similar to that of the diffuse reflector itself.

6. Time-Lagged Covariance and Temporal Spectrum of Scintillations

For this development we assume that the fields are jointly gaussian at the receiver and consequently

$$C_I(\bar{p}_1,\bar{p}_2,\tau) = |\Gamma(\bar{p}_1,\bar{p}_2,\tau)|^2 \tag{50}$$

Using the extended Huygens-Fresnel principle

$$\Gamma(\bar{p}_1,\bar{p}_2,\tau) = \left(\frac{k}{2\pi L}\right)^2 e^{\frac{ik(p_1^2-p_2^2)}{2L}} \iint d\bar{\rho}_1 d\bar{\rho}_2$$

$$\cdot <U(\bar{\rho}_1,0)U^*(\bar{\rho}_2,\tau)> \exp\left[\frac{ik}{2L}\left(\rho_1^2 - \rho_2^2 - 2\bar{p}_1 \cdot \bar{\rho}_1 - 2\bar{p}_2 \quad \bar{\rho}_2\right)\right] <\exp\left[\psi_2\left(\bar{p}_1,\bar{\rho}_1,0\right) + \psi_2^*\left(\bar{p}_2,\bar{\rho}_2,\tau\right)\right]> \tag{51}$$

Due to the diffuse target

$$<U(\bar{\rho}_1,0)U^*(\bar{\rho}_2,\tau)> = <U(\bar{\rho}_1,0)U^*(\bar{\rho}_2,\tau)>\delta(\bar{\rho}_1-\bar{\rho}_2) \tag{52}$$

Using (52) in (51) and utilizing the extended Huygens-Fresnel principle to express the fields incident on the target, the time delayed mutual coherence function at the receiver becomes for the focused case

$$\Gamma(\bar{p}_1,\bar{p}_2,\tau) = \left(\frac{k}{2\pi L}\right)^4 U_o^2\, e^{\frac{ik}{2L}\left(p_1^2-p_2^2\right)} \iiint d\bar{r}_1 d\bar{r}_2 d\bar{\rho}_1$$

$$\cdot \exp\left[-\frac{ik}{L}\,\bar{\rho}_1(\bar{p}_1-\bar{p}_2) - \frac{\left(r_1^2+r_2^2\right)}{2\alpha_o^2} - \frac{ik}{2L}\left(r_1^2-r_2^2\right)\right.$$

$$\left. + \frac{ik}{2L}(\bar{\rho}_1-\bar{r}_1)^2 - \frac{ik}{2L}(\bar{\rho}_1-\bar{r}_2)^2\right] \left\langle \exp\left[\psi_1(\bar{\rho}_1,\bar{r}_1,0)\right.\right.$$

$$\left.\left. + \psi_1^*(\bar{\rho}_1,\bar{r}_2,\tau)\right]\right\rangle \left\langle \exp\left[\psi_2(\bar{p}_1,\rho_1,0) + \psi_2^*(\bar{p}_2,\bar{\rho}_1,\tau)\right]\right\rangle \tag{53}$$

The first ensemble average in (53) corresponds to the time delayed mutual coherence function for spherical waves originating at two points $\bar{r}_1$ and $\bar{r}_2$ in the transmitter plane and propagating to a single point $\bar{\rho}_1$ in the target plane. The second ensemble average corresponds to the time delayed mutual coherence function for a spherical wave originating at the point $\bar{\rho}_1$ in the target plane and propagating to two points $\bar{p}_1$ and $\bar{p}_2$ in the receiver plane. Performing the integrations in (53) it becomes

$$\Gamma(\bar{p},\tau) = \left(\frac{k}{2\pi L}\right)^2 \left(\frac{k}{L}\right)^2 U_o^2\, \frac{\alpha_o^2}{2} \int d\bar{\rho} \int_0^\infty r\,dr\, J_o\!\left(\frac{k}{L}\rho r\right)$$

$$\cdot \exp\left[-\frac{r^2}{4\alpha_o^2} - i\frac{k}{L}\,\bar{p}\cdot\bar{\rho}\right] F(\bar{r},0,\tau)F(0,\bar{p},\tau)$$

$$= \frac{1}{2\pi}\left(\frac{k}{L}\right)^2 |U_o|^2\, \frac{\alpha_o^2}{2}\, e^{-\frac{p^2}{4\alpha_o^2} + \frac{ik}{2L}\left(p_1^2-p_2^2\right)} F(\bar{p},0,\tau)F(0,\bar{p},-\tau) \tag{54}$$

where the time delayed mutual coherence functions $F(\bar{p},0,\tau)$ and $F(0,\bar{p},\tau)$

can be obtained from the mutual coherence function for a spherical wave by invoking Taylor's Hypothesis.[5]

This mutual coherence function is given by[2]

$$F(\bar{r},\bar{\rho}) = \exp\left[-2.91\, Lk^2 \int_0^1 C_n^{\,2}(w)\,|w\bar{\rho} + (1-w)\bar{r}|^{5/3}\, dw\right] \tag{55}$$

where $\bar{r}$ is the transmitter aperture vector $\bar{r}_1-\bar{r}_2$, $\bar{\rho}$ is the target aperture vector $\bar{\rho}_1-\bar{\rho}_2$ and w is the distance from the source to the field point normalized by the total path length L. For the uniform turbulence case, we note that $k^2LC_n^{\,2} \sim \rho_o^{-5/3}$. The time delayed mutual coherence function[11] can be obtained from (55) by replacing $w\bar{\rho}$ by $w\bar{\rho} - \bar{V}(w)\tau$, where $\bar{V}(w)$ is the transverse wind:

$$F(\bar{r},\bar{\rho},\tau) = \exp\left[-2.91\, Lk^2 \int_0^1 C_n^{\,2}(w)\,|w\bar{\rho} - \bar{V}(w)\tau + (1-w)\bar{r}|^{5/3}\, dw\right] \tag{56}$$

and consequently

$$\underset{\text{To target}}{F(\bar{p},0,\tau)} = \exp\left[-2.91 \cdot Lk^2 \int_0^1 C_n^{\,2}(w)\,|(1-w)\bar{p} - \bar{V}(w)\tau|^{5/3}\, dw\right] \tag{57}$$

and

$$\underset{\text{From target}}{F(0,\bar{p},\tau)} = \exp\left[-2.91\, Lk^2 \int_0^1 C_n^{\,2}(z)\,|z\bar{p} - \bar{V}(z)\tau|^{5/3}\, dz\right] \tag{58}$$

where z = (1-w).

The normalized, time delayed covariance function for the focused case and an arbitrary distribution of $C_n^{\,2}$ along the path is thus given by

$$\underset{\text{focused}}{C_{I_N}(\bar{p},\tau)} = e^{-\frac{p^2}{2\alpha_o^{\,2}}} \exp\left[-5.82\, Lk^2 \int_0^1 C_n^{\,2}(w)\,|(1-w)\bar{p} - \bar{V}(w)\tau|^{5/3}\, dw\right] \tag{59}$$

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11. R.S. Lawrence, G.R.Ochs, and S. F. Clifford, "Use of Scintillations to Measure Average Wind Across a Light Beam", Applied Optics, 11, 239, February 1972.

For the collimated case, a further multiplicative factor

$$\left(\exp\left[-\frac{1}{2}p^2\left(\frac{k\alpha_o}{2L}\right)^2\right]\right)$$

completes the expression.

The temporal spectral density of irradiance can be obtained from (59). Letting $\vec{p} = 0$ and assuming uniform turbulence and crosswind, (59) becomes

$$C_{I_N}(0,\tau) = e^{-5.82\, Lk^2 C_n^2 |\bar{V}|^{5/3} \tau^{5/3}} = e^{-10.67\left|\frac{\bar{V}}{\rho_o}\right|^{5/3} \tau^{5/3}} \tag{60}$$

Taking the Fourier transform of (60) yields the spectral density:

$$S_I(\omega) = 2\int_0^\infty e^{-10.67\left|\frac{\bar{V}}{\rho_o}\right|^{5/3}\tau^{5/3}} \operatorname{Cos}(\omega\tau)\, d\tau$$

$$= \frac{6}{5}\frac{\rho_o}{|\bar{V}|\,(10.67)^{3/5}}\left[\Gamma(3/5) - \frac{x^2}{2!}\frac{\Gamma(9/5)}{(10.67)^{6/5}} + \ldots\right.$$

$$\left. + \ldots (-1)^{n-1}\frac{x^{2(n-1)}}{2(n-1)!}\frac{\Gamma\left(\frac{3}{5}+\frac{6}{5}(n-1)\right)}{(10.67)^{6(n-1)/5}} + \ldots\right] \tag{61}$$

where

$$x = \frac{\omega\rho_o}{|\bar{V}|}$$

The normalized spectral density is plotted versus the parameter (x) in Figure 2.

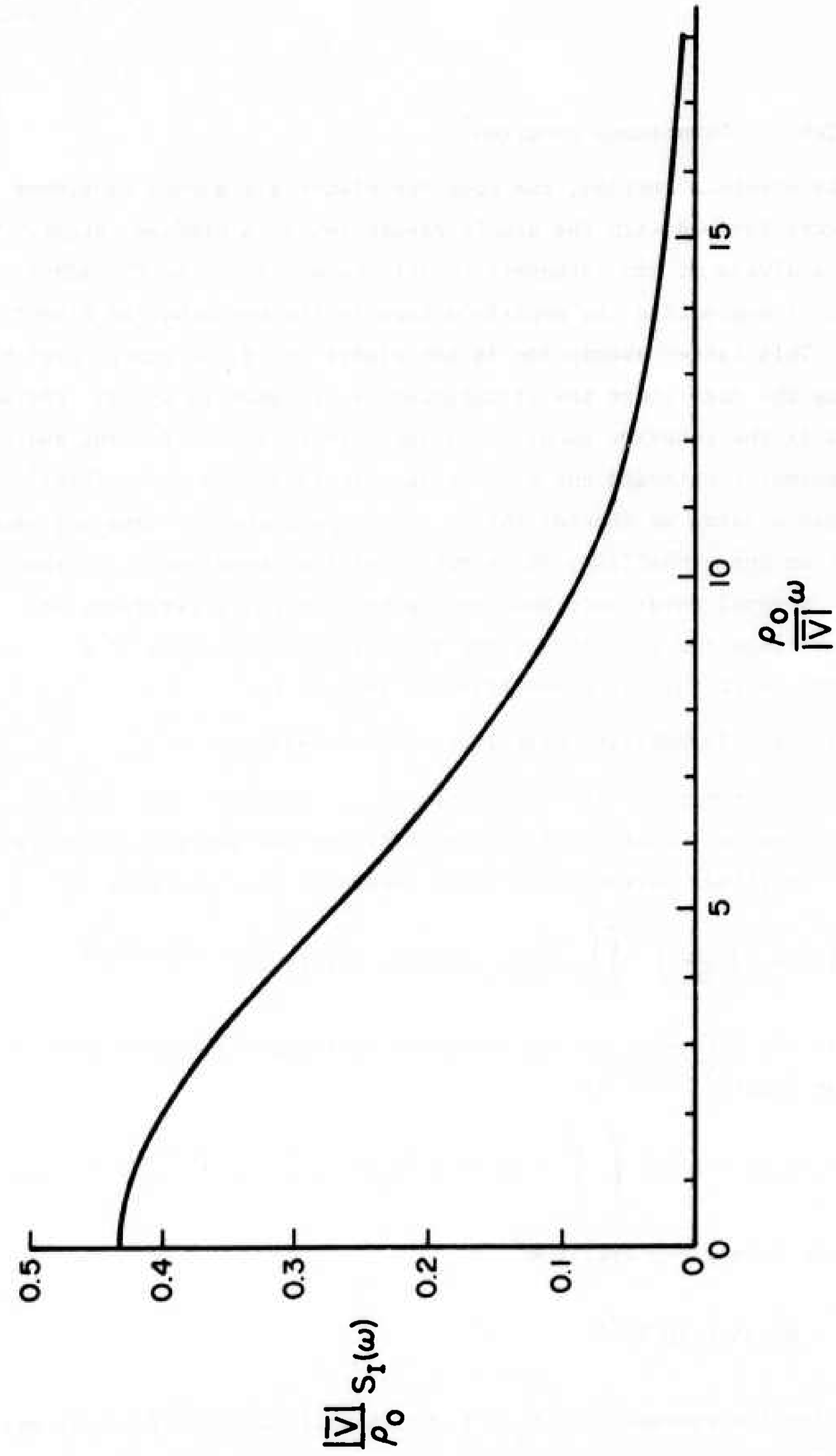

Figure 2. Normalized temporal spectrum of irradiance scintillations for a focused beam, diffuse target, and jointly-gaussian field. The frequency (ω) and spectrum (S_I) are normalized by the uniform transverse wind speed $|\bar{v}|$ and the turbulence coherence radius ρ_o as shown.

II.B. The General Covariance Function

In the previous section, the mean irradiance and mutual coherence function were derived with the simple assumption of a diffuse target. However, the analysis of the intensity covariance was based on the added assumption of neglecting the amplitude term in the perturbation Green's function. This latter assumption is not always valid, as can be seen by considering the case where the illuminated target spot is small: the scintillations at the receiver should then approximate those of point source, with log normal statistics and a covariance scale on the order $(L/k)^{1/2}$.

In this section we include the amplitude perturbation term and examine the effect on the probability distribution of the irradiance. We then derive the general covariance function, with physical interpretation. The actual conditions for validity of the simpler result of Sec. II.A will also be discussed, with further elaboration in Sec. II.E.

1. Moments and Probability Distribution of Irradiance

The second moment of irradiance can be written from Eqs. (14) and (17) and the mutual coherence function (15). We now generalize the latter to include amplitude perturbation terms (Appendix A), resulting in

$$<I^2(\bar{p})> = 2\left(\frac{k}{2\pi L}\right)^4 \iint d\bar{\rho}_2 d\bar{\rho}_4 \; <I(\bar{\rho}_2)><I(\bar{\rho}_4)> \; e^{4C_\chi}\left(|\bar{\rho}_2-\bar{\rho}_4|\right) \tag{61}$$

where C_χ is the point-source log amplitude covariance function given in the first-order theory[5,12,13] by

$$C_\chi(|\bar{\rho}_2-\bar{\rho}_4|) = 4\pi^2k^2 \int_0^\infty \int_0^L u\Phi(u) J_0\left(\frac{|\bar{\rho}_2-\bar{\rho}_4|}{L} us\right) \sin^2\left[\frac{u^2 s(L-s)}{2kL}\right] ds\,du \tag{62}$$

and Φ is the Kolmogorov spectrum[5]

$$\Phi(u) = 0.033\, C_n^{\;2} u^{-11/3} \tag{63}$$

We assume for the present that $C_\chi << 1$, which will be true[5] for both weak

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12. V.I. Tatarski, <u>The Effects of the Turbulent Atmosphere on Wave Propagation</u>, National Technical Information Service (#TT-68-50464), 1971.
13. R. W. Lee and J. C. Harp, "Weak Scattering in Random Media, with Applications to Remote Probing", Proc. IEEE, <u>57</u>, 375, April 1969.

and saturated scintillations (weak and strong turbulence scattering respectively). It will not be true for the intermediate case, however, and we will lift the restriction in calculating the covariance below. Also, we point out that the function C_χ has been derived phenomenologically for the saturated or strong scattering case by Yura and Clifford[14-16] so that in principle we are not limited to the first-order expression (62).

We generalize (61) to the nth moment:

$$\langle I^n(\bar{p})\rangle = n!\left(\frac{k}{2\pi L}\right)^{2n} \int\cdots\int d\bar{\rho}_1\ldots d\bar{\rho}_n \left[\prod_{i=1}^{n} \langle I(\bar{\rho}_i)\rangle\right] \exp\left[2\sum_{i\neq j}^{n} C_{\chi_{ij}}\right] \tag{64}$$

We write this as

$$\langle I^n(\bar{p})\rangle = n!\langle I(\bar{p})\rangle^n F_n \tag{65}$$

where

$$F_1 = 1$$

$$F_2 = \frac{\iint d\bar{\rho}_2 d\bar{\rho}_4 \langle I(\bar{\rho}_2)\rangle\langle I(\bar{\rho}_4)\rangle\, e^{4C_\chi(|\bar{\rho}_2-\bar{\rho}_4|)}}{\left[\int d\bar{\rho}_2\langle I(\bar{\rho}_2)\rangle\right]^2}$$

$$F_n = \frac{\int\cdots\int d\bar{\rho}_1 d\bar{\rho}_2\ldots d\bar{\rho}_n\left[\prod_{i=1}^{n}\langle I(\bar{\rho}_i)\rangle\right]\exp\left[2\sum_{i\neq j}^{n} C_\chi(|\bar{\rho}_i-\bar{\rho}_j|)\right]}{\left[\int d\bar{\rho}\langle I(\bar{\rho})\rangle\right]^n} \tag{66}$$

Our assumption on C_χ yields

$$e^{4C_\chi(|\bar{\rho}_2-\bar{\rho}_4|)} = 1+4C_\chi(|\bar{\rho}_2-\bar{\rho}_4|) \tag{67}$$

We thus simplify F_2 to

$$F_2 = 1+\frac{4\iint d\bar{\rho}_2 d\bar{\rho}_4\langle I(\bar{\rho}_2)\rangle\langle I(\rho_4)\rangle C_\chi(|\bar{\rho}_2-\bar{\rho}_4|)}{\left[\int d\bar{\rho}_2\langle I(\bar{\rho}_2)\rangle\right]^2} \tag{68}$$

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14. H. T. Yura, "Physical Model for Strong Optical-Amplitude Fluctuations in a Turbulent Medium", JOSA, 64, 59, January 1974.
15. S. F. Clifford, G. R. Ochs, and R. S. Lawrence, "Saturation of Optical Scintillation by Strong Turbulence", JOSA, 64, 148, February 1974.
16. S. F. Clifford and H. T. Yura, "Equivalence of Two Theories of Strong Optical Scintillation", JOSA, 64, 1641, December 1974.

For the denominator for the focused case, we write from Eq. (10):

$$<I(\bar{\rho})> = \left(\frac{k}{L}\right)^2 |U_o|^2 \frac{\alpha_o^{\,2}}{2} \int_0^\infty r dr J_o\left(\frac{k}{L}\rho r\right) e^{-\frac{r^2}{4\alpha_o^{\,2}} - \left(\frac{r}{\rho_o}\right)^{5/3}} \tag{69}$$

which integrates to

$$\int <I(\bar{\rho})> d\bar{\rho} = 2\pi |U_o|^2 \frac{\alpha_o^{\,2}}{2} \tag{70}$$

The integral in the numerator of (68) is straightforward but laborious, We state here the result for F_2:

$$F_2 = 1+(4\pi)^2 k^2 \int_0^\infty u du \int_0^L ds\, e^{-\frac{u^2 s^2}{2\alpha_o^{\,2} k^2} - 2\left(\frac{us}{k\rho_o}\right)^{5/3}} \Phi(u) \sin^2\left[\frac{u^2 s(L-s)}{2kL}\right] \tag{71}$$

where the approximation relates to the assumption (67) on $C\chi$. We then have the second moment from Eq. (65):

$$<I^2(\bar{\rho})> = 2<I(\bar{\rho})>^2 F_2 \tag{72}$$

and the n^{th} moment is

$$<I^n> = n!<I>^n \left\{1+ \frac{n(n-1)}{2} (F_2-1)\right\} \tag{73}$$

Hence nonunity F_2 represents the departure from an exponential intensity distribution.

We finally derive the probability distribution using

$$<e^{-sI}> = 1-s<I>+ \frac{s^2<I^2>}{2!} - \frac{s^3<I^3>}{3!} + \ldots \tag{74}$$

We let $<I> \equiv I_o$ and find the probability as

$$P_I(I) = \frac{1}{I_o} e^{-\frac{I}{I_o}} \quad 1 + (F_2-1)\left[1- \frac{2I}{I_o} + \frac{I_2}{2I_o^{\,2}}\right] \tag{75}$$

so that we have a first-order correction to the exponential distribution.

We note that F_2 (and F_n) will be unity for weak turbulence and for very strong turbulence ($\rho_o \to 0$). Also, F_2 will be unity for a small focused source (α_o), which physically relates to a large target spot. Conversely, F_2 will depart from unity and therefore show the effect of the amplitude perturbation term for the case of intermediate turbulence and a large source (small target spot). Physically this is the case of a quasi-point-source attempting to scintillate in the usual manner for a point source in the first-order theory,[5] but nevertheless interacting with the speckles created by the diffuse target.

The exponential distribution applies for the weak turbulence case simply because the behavior is that of a diffuse source in a vacuum (speckles). It applies to a large target-spot, implying that the phase-perturbation-dominance assumption of Section II.A is then applicable, i.e. that the target (vacuum) speckle mechanism dominates. It applies for strong turbulence because the field again has the nature of that from a diffuse source (atmospheric speckle ρ_o). These considerations will be clarified below and in Section II.E.

2. Covariance and Variance of Intensity

We now derive the general covariance function, dropping the requirement that $C_\chi \ll 1$.

The covariance is the second term of Eq. (18):

$$C_I(\bar{p}_1,\bar{p}_2) = \left(\frac{k}{2\pi L}\right)^4 \iint d\bar{\rho}_2 d\bar{\rho}_4 \; \langle I(\bar{\rho}_2)\rangle \langle I(\bar{\rho}_4)\rangle \; e^{\frac{ik}{L}(\bar{p}_1-\bar{p}_2)(\bar{\rho}_1-\bar{\rho}_2)} \; H(\bar{\rho},\bar{p}) \quad (76)$$

where

$$\bar{\rho} = \bar{\rho}_2 - \bar{\rho}_4 \text{ and } \bar{p} = \bar{p}_1 - \bar{p}_2 \; .$$

The full coherence function replacing Eq. (19) is (Appendix A)

$$H(\bar{\rho},\bar{p}) = e^{\left\{-2\left(\frac{p}{\rho_o}\right)^{5/3} - 2\left(\frac{\rho}{\rho_o}\right)^{5/3} + \frac{\left[p^{8/3}-\rho^{8/3}\right]}{\rho_o^{5/3}}\left[\frac{1}{|\bar{p}+\bar{\rho}|} + \frac{1}{|\bar{p}-\bar{\rho}|}\right] + 2C_\chi(\bar{\rho},\bar{p}) + 2C_\chi(\bar{\rho},-\bar{p})\right\}} \quad (77)$$

where[17]

$$C_\chi(\bar{p},\bar{p}) = 0.132\pi^2 k 2 C_n^{\,2} L \int_0^1 dt \int_0^\infty du\, u^{-8/3} \sin^2\left[\frac{u^2 Lt(1-t)}{2k}\right]$$

$$\cdot J_o\left[u\left|t\bar{p} + (1-t)\rho\right|\right] \tag{78}$$

The latter expression (78) assumes that multiple scattering (saturation) does not apply but can be modified for such a case.[15] The irradiance at the target $\langle I(\bar{\rho})\rangle$ is given by (69) for the focused case and with an additional $(k\alpha_o/2L)^2$ term in the exponent for the collimated case as before.

We again omit tedious algebra and integrations and state the result for the focused case:

$$C_I(\bar{p}) = \left(\frac{1}{2\pi}\right)^3 \left(\frac{k}{L}\right)^6 |U_o|^4 \left(\frac{\alpha_o}{2}\right)^2 \epsilon^{-2\left(\frac{p}{\rho_o}\right)^{5/3}} \int_0^\infty r_2 dr_2\, \ell^{-\frac{r^2}{2\alpha_o^{\,2}} - 2\frac{r_2}{\rho_o}^{5/3}}$$

$$\cdot \int d\bar{\rho} V_o\left(\frac{k}{L} r_2 \rho\right) e^{\frac{ik}{L}\bar{\rho}\cdot\bar{p}}\, H(\bar{\rho},\bar{p}) \tag{79}$$

The variance is given by

$$C_I(o) = \sigma_I^{\,2} = \left(\frac{1}{2\pi}\right) \left(\frac{k}{L}\right)^6 |U_o|^4 \left(\frac{\alpha_o}{2}\right)^2 \int_0^\infty r_2 dr_2\, e^{-\frac{r_2^{\,2}}{2\alpha_o^{\,2}} - \frac{2 r_2^{\,5/3}}{\rho_o^{\,5/3}}}$$

$$\cdot \int_0^\infty \rho d\rho J_o\left(\frac{k}{L} r_2 \rho\right) e^{4C_\chi(\rho)} \tag{80}$$

These general results involve five-fold integrals (more in the saturated case[15]) and have not been numerically evaluated. A comparison with Eqs. (26) and (28) show that the amplitude perturbation term simply introduces the C_χ terms in Eq. (77), i.e. the log amplitude covariance for a point

17. T.Wang, S.F.Clifford, and G.R.Ochs, "Wind and Refractive Turbulence Sensing Using Crossed Laser Beams", Applied Optics, 13, 2602, November 1974.

source. A qualitative interpretation will be given below.

In order to compare these results with those of Section II.B.1, we again let $C_\chi << 1$, and we find that (80) and (63) yield

$$\sigma^2_{I_N} = \frac{\sigma^2_I}{\bar{I}^2}$$

$$= 1 + 0.528\pi^2 k^2 C_n^2 L \int_0^1 dt \int_0^\infty du\, u^{-8/3} \exp\left\{- \frac{1}{2\alpha_o{}^2}\left[\frac{u}{k} L(1-t)\right]^2\right.$$

$$\left. - \frac{2}{\rho_o{}^{5/3}}\left[\frac{u}{k} L(1-t)\right]^{5/3}\right\} \sin^2\left[\frac{u^2 Lt(1-t)}{2k}\right] \tag{81}$$

This is consistent with Eq. (72), and again shows the departure from the unity normalized variance obtained when C_χ cannot be taken as zero.

3. Further Remarks on Probability Distributions and Limiting Conditions

A systematic, quantitative description of parameter realms will be given in Section II.E. However, we can indicate here the types of considerations involved. Let us write the general coherence function (77) as

$$H(\bar{\rho},\bar{p}) = e^{-2\left(\frac{p}{\rho_o}\right)^{5/3}} H'(\bar{\rho},\bar{p}) \tag{82}$$

where

$$H'(\bar{\rho},\bar{p}) = e^{-2\left(\frac{\rho}{\rho_o}\right)^{5/3} + \frac{|p^{8/3}-\rho^{8/3}|}{\rho_o{}^{5/3}}\left[\frac{1}{|\bar{p}+\bar{\rho}|} + \frac{1}{|\bar{p}-\bar{\rho}|}\right] + 2C_\chi(\bar{\rho},\bar{p}) + 2C_\chi(\bar{\rho},-\bar{p})} \tag{83}$$

From the preceding discussions, we know that non-negligible C_χ terms relate to a nongaussian field (irradiance is not exponentially distributed, and that $H'=1$ corresponds to a jointly gaussian field. The amplitude perturbation effects (C_χ) will be influential under the following conditions (see Eqs. (79) and (83)):

A. Turbulence is moderate or weak (i.e. $\rho_o > (L/k)^{1/2}$, and

B. The source is large ($\alpha_o > (L/k)^{1/2}$).

These conditions can be reasoned from Eq. (79) by considering the large range of r_2 in the exponential, leading to a small range of ρ from the

Bessell function, and the corresponding influence of C_χ in (83) on the result of the ρ integration. Physically, this corresponds to nonsaturated turbulence, with a near-field target and small spot acting as a quasi-point-source.

Conversely, if α_o is small, the range of r_2 is limited, and that of ρ expanded such that C_χ does not make an important contribution to the ρ integral. This means that a large target spot results in dominance of the phase perturbation terms. In fact, if $\rho_o >> \alpha_o$, H' can be taken as unity, the field is jointly gaussian, and the covariance is given by Eq. (47). Finally, if turbulence is strong $(\rho_o < (L/k)^{\frac{1}{2}})$, which corresponds to the multiple scattering realm, then with a large or small source the ρ-integral will be controlled by the phase terms in (83). Thus the phase perturbation dominates in strong scattering; the field will be jointly gaussian except in the transition region $\rho_o \approx \alpha_o$, where it is simply gaussian.

4. Further Discussion of the Covariance Function and Scales

The qualitative behavior of the covariance curve, with the attendant covariance scales or scintillation patch size, can be described as follows. Suppose that we have a large, focused transmitter (near-field target), with weak turbulence $(\alpha_o, \rho_o > (L/k)^{\frac{1}{2}})$. The scintillations will be essentially those of a point source (unsaturated):[5]

$$\sigma_\chi^2 = 0.124\ C_n^2 k^{7/6}\ L^{11/6} \tag{84}$$

for the log amplitude variance. The covariance curve for irradiance will be as shown in Figure 3a, where the covariance scale is proportional to $(L/k)^{\frac{1}{2}}$. Now let the turbulence strength increase (ρ_o decrease) until ρ_o is comparable to $(L/k)^{\frac{1}{2}}$; the log amplitude variance peaks at approximately 0.6 before saturating,[5] and a new covariance scale (ρ_o) appears as conceptualized in Figure 3b. Finally, let the turbulence become stronger yet; the normalized irradiance variance is unity and the covariance curve is as given in Figure 3c. The third possible covariance scale, α_o itself, will only predominate when α_o is small (far-field target). In the latter case only, the speckles will not be transported by the transverse wind.

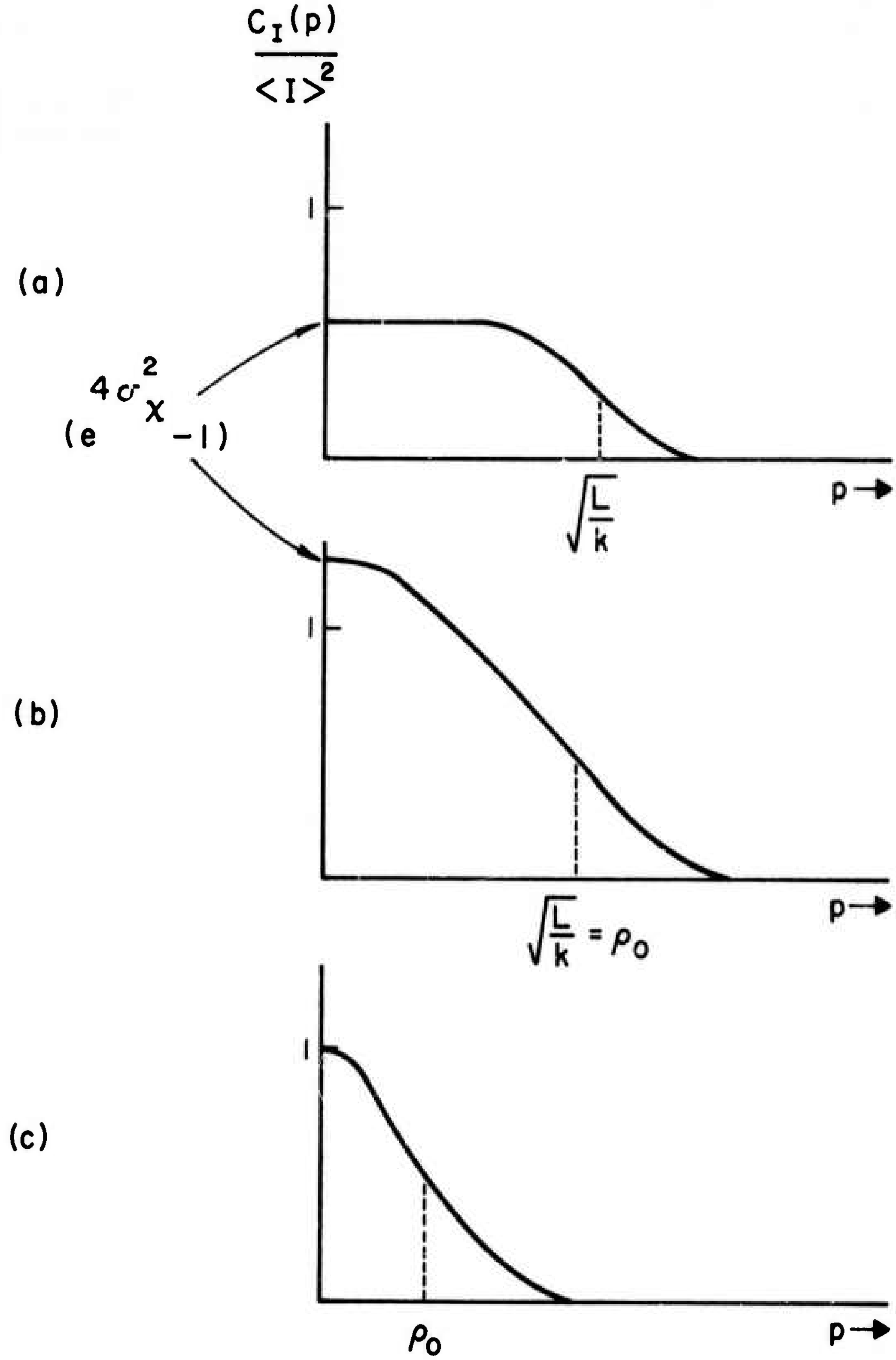

Figure 3. Conceptual normalized function of irradiance for a large, focused source and varying turbulence strength. $\sigma_\chi{}^2$ denotes the log amplitude variance for a point source.

A. Weak turbulence ($\rho_o >> (L/k)^{1/2}$)

B. Transition region ($\rho_o \approx (L/k)^{1/2}$

C. Strong turbulence or saturation ($\rho_o << (L/k)^{1/2}$.

This will also be true for a large target spot arising from defocusing or collimation, such that the associated speckles are small.

II.C. Target Glints

As a preliminary examination of the effects of target structure, we present in this section an analysis of the mean receiver irradiance in the presence of "glints" (specular reflectors). We assume that the target is otherwise diffuse, and that phase perturbations predominate. The transmitter is again a TEM_{00} focused or collimated source.

We again start with the field at the receiver as described by Eqs. (1,2). The field reflected at the target is represented as

$$U(\bar{\rho}) = U_{diffuse}(\bar{\rho}) + U_{glint}(\bar{\rho}) = U_d(\bar{\rho}) + U_g(\bar{\rho}) \tag{85}$$

where

$$U_{glint} = \sum_{m=1}^{M} U_{incident}(\bar{\rho}_m)\, a_m \exp\left\{\frac{-(\bar{\rho}-\bar{\rho}_m)^2}{\Delta\rho_m^{\ 2}}\right\}$$

ρ_m = position of m^{th} glint

$\Delta\rho_m$ = width of m^{th} glint

a_m = complex strength of m^{th} glint

Then the irradiance is

$$I(\bar{p}) = \left(\frac{k}{2\pi L}\right)^2 \iint \left[U_d(\bar{\rho}_1) + U_g(\bar{\rho}_1)\right]\left[U_d^*(\bar{\rho}_2) + U_g^*(\bar{\rho}_2)\right]$$

$$\cdot \exp\left\{\frac{ik}{2L}\left[(\bar{\rho}_1-\bar{p})^2 - (\bar{\rho}_2-\bar{p})^2\right] + \psi_2(\bar{\rho}_1,\bar{p}) + \psi_2^*(\bar{\rho}_2,\bar{p})\right\} d\bar{\rho}_1, d\bar{\rho}_2$$

$$= \left(\frac{k}{2\pi L}\right)^2 \iint \left[U_d(\bar{\rho}_1) + U_g(\bar{\rho}_1)\right]\left[U_d^*(\bar{\rho}_2) + U_g^*(\bar{\rho}_2)\right]$$

$$\cdot \exp\left\{\frac{ik}{2L}\left[\rho_1^2 - \rho_2^2 - 2(\bar{\rho}_1-\bar{\rho}_2)\bar{p}\right] + \psi_2(\bar{\rho}_2,\bar{p}) + \psi_2^*(\bar{p}_2,\bar{p})\right\} d\bar{\rho}_1, d\bar{\rho}_2 \tag{86}$$

We then simplify the field at the target:

$$< \left[U_d(\bar{\rho}_1)+U_g(\bar{\rho}_1)\right]\left[U_d^*(\bar{\rho}_2)+U_g^*(\rho_2)\right] >$$

$$= <U_d(\rho_1)U_d^*(\bar{\rho}_2)> + <U_d(\bar{\rho}_1)U_g^*(\bar{\rho}_2)> + <U_g(\bar{\rho}_1)U_d^*(\bar{\rho}_2)>$$

$$+ <U_g(\bar{\rho}_1)U_g^*(\rho_2)> \tag{87}$$

Since

$$<U_d(\bar{\rho}_1)U_g^*(\bar{\rho}_2)> = <U_d(\bar{\rho}_1)><U_g^*(\rho_2)> = 0$$

$$<U_g(\bar{\rho}_1)U_d^*(\bar{\rho}_2)> = <U_g(\bar{\rho}_1)><U_d^*(\bar{\rho}_2)> = 0$$

$$<U_d(\bar{\rho}_1)U_d^*(\bar{\rho}_2)> = <I(\bar{\rho}_1)> \delta\ (\bar{\rho}_1-\bar{\rho}_2)$$

$$< e^{\psi_2(\bar{\rho}_1,\bar{p})+\psi_2^*(\bar{\rho}_2,\bar{p})} >= e^{-\left(\frac{|\bar{\rho}_1-\bar{\rho}_2|}{\rho_o}\right)^{5/3}} \tag{88}$$

we have

$$<I(\bar{p})> = <I(\bar{p})>_d + <I(\bar{p})>_g \tag{89}$$

where $<I(\bar{p})>_d$ is the diffuse term analyzed in Section II,A and $<I(\bar{p})>_g$ is the glint term:

$$<I(\bar{p})>_g = \left(\frac{k}{2\pi L}\right)^2 \iint d\bar{\rho}_1 d\bar{\rho}_2\ <U_g(\bar{\rho}_1)U_g^*(\bar{\rho}_2)>$$

$$\cdot\ \exp\left\{\frac{ik}{2L}\left[\rho_1^2-\rho_2^2-2(\bar{\rho}_1-\bar{\rho}_2)\cdot\bar{p}\right] - \frac{|\bar{\rho}_1-\bar{\rho}_2|^{5/3}}{\rho_o^{5/3}}\right\} \tag{90}$$

We now describe the complex correlation of the glint field at the target:

$$<U_g(\rho_1)U_g^*(\bar{\rho}_2)> = \sum_{m=1}^{M} |a_m|^2\ <I(\bar{\rho}_m)>^2 \exp\left\{-\frac{\rho_1^2+\rho_2^2-2\bar{\rho}_m\cdot(\bar{\rho}_1+\bar{\rho}_2)+2\rho_m^2}{\Delta\rho_m^2}\right\}$$

$$+ \sum_{m_1 \neq m_2}^{M} a_{m_1} a_{m_2}\ <U(\bar{\rho}_{m_1})U^*(\bar{\rho}_{m_2})>$$

$$.\exp \; - \left\{ \frac{\rho_1^2 - 2\bar{\rho}_{m_1} \cdot \bar{\rho}_1 + \rho_{m_1}^{\;2}}{\Delta\rho_{m_1}^{\;2}} - \frac{\rho_2^2 - 2\bar{\rho}_{m_2} \cdot \bar{\rho}_2 + \rho_{m_2}^{\;2}}{\Delta\rho_{m_2}^{\;2}} \right\} \tag{91}$$

where for the focused case

$$\langle I(\bar{\rho}_m) \rangle = \left(\frac{k}{L}\right)^2 |U_o|^2 \frac{\alpha_o^{\;2}}{2} \int_0^\infty r\,dr\, J_o\left(\frac{k}{L}\rho_m r\right) e^{-\frac{r^2}{4\alpha_o^{\;2}} - \left(\frac{r}{\rho_o}\right)^{5/3}} \tag{92}$$

The corresponding expression for the collimated case has an additional $(k\alpha_o/2L)^2$ term in the exponent. The coherence function can be readily shown to be

$$\langle U(\rho_{m_1}) U^*(\rho_{m_2}) \rangle = \frac{1}{2\pi}\left(\frac{k}{L}\right)^2 |U_o|^2 \frac{\alpha_o^2}{2} e^{-\left[\frac{\alpha_o}{2} \frac{k}{L} |\bar{\rho}_{m_1} - \bar{\rho}_{m_2}|\right]^2 - \frac{ik}{2L}\left(\rho_{m_1}^{\;2} - \rho_{m_2}^{\;2}\right)}$$

$$\cdot \int d\bar{r}\; e^{-\frac{r^2}{4\alpha_o^2} - \frac{ik}{2L} \bar{r} \cdot (\bar{\rho}_{m_1} + \bar{\rho}_{m_2})} \cdot F(\bar{r}, \bar{\rho}_{m_1} - \bar{\rho}_{m_2}) \tag{93a}$$

$$\langle U(\rho_{m_1}) U^*(\rho_{m_2}) \rangle = \frac{1}{2\pi}\left(\frac{k}{L}\right)^2 |U_o|^2 \frac{\alpha_o^{\;2}}{2} \int d\bar{r}\; e^{-\frac{r^2}{4\alpha_o^2} - \left[\frac{\alpha_o}{2} \frac{k}{L} |\bar{r} - (\rho_{m_1} - \rho_{m_2})|\right]^2}$$

$$e^{-\frac{ik}{2L} \bar{r} \left(\bar{\rho}_{m_1} + \bar{\rho}_{m_2}\right) - \frac{ik}{2L}\left(\rho_{m_1}^{\;2} - \rho_{m_2}^{\;2}\right)} \; F(\bar{r}, \bar{\rho}_{m_1} - \bar{\rho}_{m_2}) \tag{93b}$$

where the mutual coherence function F is

$$F(\bar{r}, \bar{\rho}_{m_1} - \bar{\rho}_{m_2}) = F(\bar{r}, \rho) = e^{-\frac{2.91}{2} C_n^{\;2} L k^2 \left[\frac{3}{8} \frac{|\rho^{8/3} - r^{8/3}|}{|\bar{\rho} - \bar{r}|}\right]} \tag{94}$$

Finally, the mean receiver irradiance (diffraction pattern) due to glints is expressed from (90,91) as

$$\langle I(\bar{p})\rangle_g = \left(\frac{k}{2\pi L}\right)^2 \sum_{m_1=1}^{M} \sum_{m_2=1}^{M} a_{m_1} a^*_{m_2} \langle U(\bar{\rho}_{m_1}) U^*(\bar{\rho}_{m_2})\rangle$$

$$\cdot \iint d\bar{\rho}_1 d\bar{\rho}_2 \exp\left\{\frac{ik}{2L}\left[\rho_1^2-\rho_2^2-2(\bar{\rho}_1-\bar{\rho}_2)\cdot\bar{p}\right] - \frac{(\bar{\rho}_1-\bar{\rho}_{m_1})^2}{\Delta\rho_{m_1}^2} - \frac{(\bar{\rho}_2-\bar{\rho}_{m_2})^2}{\Delta\rho_{m_2}^2} - \left(\frac{\rho}{\rho_o}\right)^{5/3}\right\} \tag{95}$$

where $\langle U(\bar{p}_{m_1})U^*(\bar{\rho}_{m_2})\rangle$ is given by Eqs. (93). We now show specific cases.

1. Single glint (M = 1)

From Eq. (95) we have

$$\langle I(\bar{p})\rangle_g = \left(\frac{k}{2\pi L}\right)^2 \langle I(\bar{\rho}_m)\rangle \, |a_m|^2$$

$$\cdot \iint d\bar{\rho}_1 d\bar{\rho}_2 \exp\left\{\frac{ik}{2L}\left[\rho_1^2-\rho_2^2-2(\bar{\rho}_1-\bar{\rho}_2)\cdot\bar{p}\right] - \frac{\rho_1^2+\rho_2^2-2\bar{\rho}_m\cdot(\bar{\rho}_1+\bar{\rho}_2)+2\rho_m^2}{\Delta\rho_m^2} - \left(\frac{\rho}{\rho_o}\right)^{5/3}\right\} \tag{96}$$

With the following change of variables:

$$\left.\begin{aligned} \bar{\rho}_1-\bar{\rho}_2 &= \bar{\rho} \\ \bar{\rho}_1+\bar{\rho}_2 &= 2\bar{R} \end{aligned}\right\} \quad \rho_1^2+\rho_2^2 = \frac{1}{2}(\rho^2+4R^2) \tag{97}$$

we have

$$\langle I(\bar{p})\rangle_g = \left(\frac{k}{2\pi L}\right)^2 \langle I(\bar{\rho}_m)\rangle |a_m|^2 \, e^{-2\left(\frac{\rho_m}{\Delta\rho_m}\right)^2}$$

$$\cdot \int d\bar{R} \; e^{\frac{ik}{L}\bar{\rho}\cdot\bar{R} + \frac{4\bar{\rho}_m \cdot \bar{R}}{\Delta\rho_m^2} - \frac{2R^2}{\Delta\rho_m^2}}$$

$$\cdot \int d\bar{\rho} \; e^{-\frac{ik}{L}\bar{\rho}\cdot\bar{p} - \frac{\rho^2}{2\Delta\rho_m^2} - \left(\frac{\rho}{\rho_o}\right)^{5/3}} \tag{98}$$

The first integration yields

$$2\pi \int_0^\infty R\, J_o \left[\left| \frac{k}{L} \bar{p} - \frac{i4\rho_m}{\Delta\rho_m^{\,2}} \right| R \right] e^{-2\left(\frac{R}{\Delta\rho_m}\right)^2} dR$$

$$= 2\pi \frac{\Delta\rho_m^{\,2}}{4}\, e^{-\frac{\Delta\rho_m^{\,2}}{8}\left[\left(\frac{k}{L}\, p\right)^2 - \left(\frac{4\rho_m}{\Delta\rho_m}\right)^2\right] + \frac{ik}{L}\, \bar{\rho}_m \cdot \bar{p}} \tag{99}$$

which with (98) gives

$$\langle I(\bar{p})\rangle_g = \left(\frac{k}{L}\right)^2 \langle I(\bar{\rho}_m)\rangle |a_m|^2\, \frac{\Delta\rho_m^{\,2}}{4}$$

$$\cdot \int_0^\infty \rho\, J_o\left[\frac{k}{L}\, |\bar{p}-\bar{\rho}_m|\rho\right] e^{-\frac{1}{4}\left[\frac{2}{\Delta\rho_m^{\,2}} + \frac{\Delta\rho_m^{\,2}}{2}\left(\frac{k}{L}\right)^2\right]\rho^2 - \left(\frac{\rho}{\rho_o}\right)^{5/3}} d\rho \tag{100}$$

This may be usefully normalized by setting $\bar{p} = \bar{\rho}_m$.

The result (100) may be interpreted as follows. The maximum occurs at $\bar{p} = \bar{\rho}_m$, as expected. The scale of the diffraction pattern ($|\bar{p} - \bar{\rho}_m|$) is seen from the Bessel term to be $L/k\rho_{max}$, where ρ_{max} is determined from the exponential as the smallest of $\Delta\rho_m$, $L/k\Delta\rho_m$, and ρ_o respectively. The corresponding $I(\bar{p})$ scales are $L/k\Delta\rho_m$ (diffraction from the glint), $\Delta\rho_m$ (geometric reflection term), and ρ_o (effective coherent glint size in strong turbulence); the largest will predominate in each case.

2. Arbitrary Number of Glints (M)

This involves lengthy but straightforward calculation. The result is

$$\langle I(\bar{p})\rangle_g = \sum_{i=1}^{M} \langle I(\bar{p})\rangle_{g_i} + \frac{1}{2} \sum_{i\neq j}^{M} F_{i,j}(I) \tag{101}$$

where

$$F_{i,j}(I) = \left(\frac{k}{L}\right)^2 \ \mathrm{Re}\left[\langle U(\bar{\rho}_{m_i}) U^*(\bar{\rho}_{m_j}) \rangle a_{m_i} a_{m_j} (\Delta\rho_{ij+}^{\,2}) \right.$$

$$\cdot \exp\left\{ - \frac{|\bar{\rho}_{m_j} - \bar{\rho}_{m_i}|^2}{\Delta\rho_{m_i}^{\,2} + \Delta\rho_{m_j}^{\,2}} \right\}$$

$$\cdot \int \rho d\rho \ J_o(|\bar{\gamma}|\rho) \ \exp\left\{ - \frac{1}{4}\left[\frac{1}{\Delta\rho_{ij+}^{\,2}} + \Delta\rho_{ij+}^{\,2}\left(\frac{k}{L} + \frac{i}{\Delta\rho_{ij-}^{\,2}}\right)^2 \right] \rho^2 \right.$$

$$\left. \left. - \left(\frac{\rho}{\rho_o}\right)^{5/3} \right\} \right] \tag{102}$$

and

$$\Delta\rho_{ij\pm}^{\,2} = \frac{\Delta\rho_{m_i}^{\,2} . \Delta\rho_{m_j}^{\,2}}{\Delta\rho_{m_1}^{\,2} \pm \Delta\rho_{m_2}^{\,2}}$$

$$\bar{\gamma} = \frac{k}{L}\left[\bar{p} - \left(\bar{\rho}_{m_i} + \frac{(\bar{\rho}_{m_j} - \bar{\rho}_{m_i})\Delta\rho_{m_i}^{\,2}}{\Delta\rho_{m_i}^{\,2} + \Delta\rho_{m_j}^{\,2}} \right) \right] + i\ 2\ \frac{\Delta\rho_{m_i}^{\,2}(\bar{\rho}_{m_j} - \bar{\rho}_{m_i})}{\Delta\rho_{m_i}^{\,2} + \Delta\rho_{m_j}^{\,2}} \tag{103}$$

Each pair of glints results in a cross-term bright spot in the receiver irradiance field, located at

$$\bar{p}_{ij} = \left(\bar{\rho}_{m_i} + \frac{(\bar{\rho}_{m_j} - \bar{\rho}_{m_i})\Delta\rho_{m_i}^{\,2}}{\Delta\rho_{m_i}^{\,2} + \Delta\rho_{m_j}^{\,2}} \right) \tag{104}$$

This is illustrated in Figure 4.

II.D. Incoherent Illumination

We now depart from the case of TEM_{00} laser illumination and briefly consider the non-monochromatic case, which might include that of a highly multimode laser. In principle, all of the expressions of the preceding sections can be generalized with the utilization of a

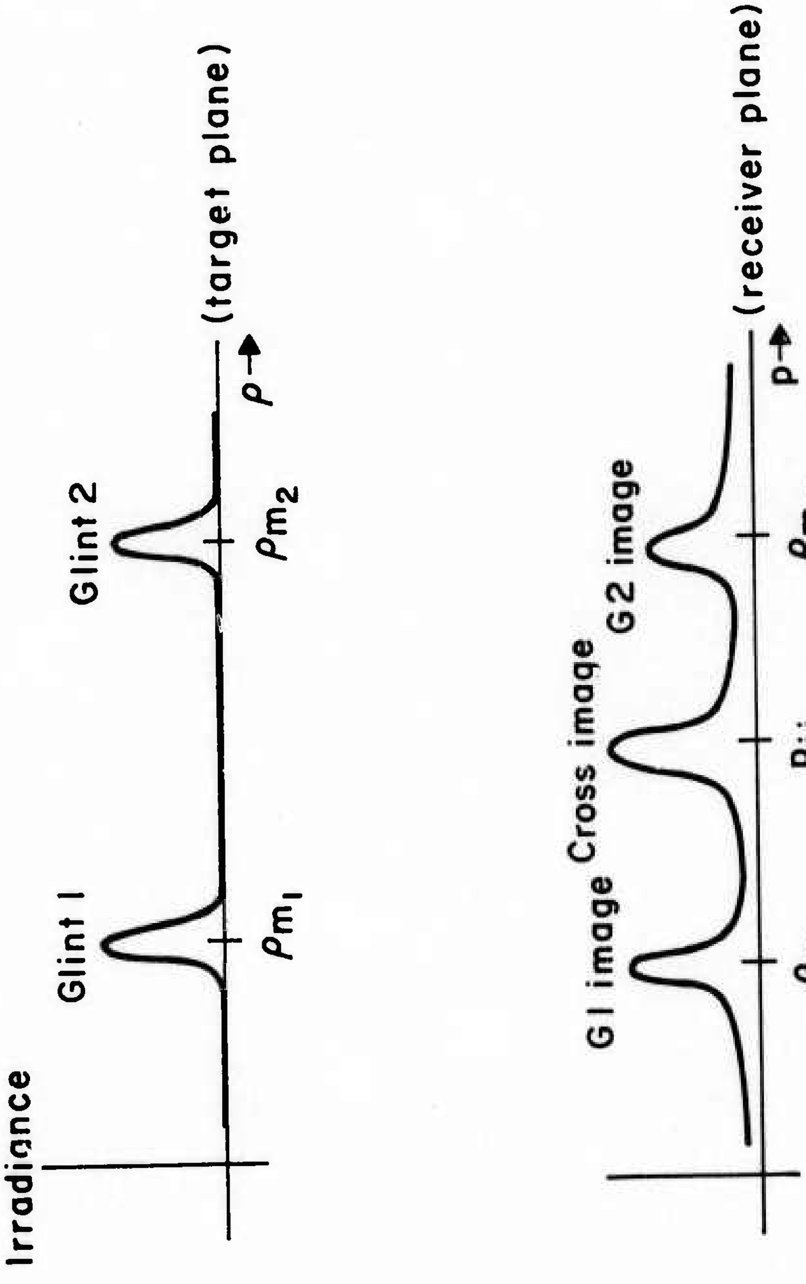

Figure 4. Irradiance distribution for two glints.

Fourier transform or partial-coherence representation of the source. As the source becomes monochromatic, the first-order effect will be to wash out the speckles (α_0 and ρ_0 terms in Eq. 47). This will obviate the influence of the phase-perturbation terms (Secs. II.A,B) and the remaining scintillations will be due to the amplitude-perturbation term and to geometrical considerations, as will be seen below. As the spectral width of the source grows further, scintillations e.g. for a point source will be further smoothed by "spectral diversity",[18] but this is a weak effect requiring a substantial fractional spectral width.

The situation may be simply analyzed if we assume a completely incoherent but quasi-monochromatic target spot. We start with the intensity correlation function at the receiver from Eq. (14), and utilize delta-function correlation of $U(\rho_1)$ with $U^*(\rho_2)$ and also $U(\rho_3)$ with $U^*(\rho_4)$ to write immediately

$$<I(\bar{p}_1)I(\bar{p}_2)> = \left(\frac{k}{2\pi L}\right)^4 \iint d\bar{\rho}_1 d\bar{\rho}_3 \; |U(\bar{\rho}_1)|^2 |U(\bar{\rho}_3)|^2$$

$$\cdot \exp \left\{ 4\, C\chi(\rho_3-\rho_1,p) \right\} \tag{105}$$

As expected, the target-speckle (α_0) and atmospheric speckle (ρ_0) terms disappear, leaving the log amplitude covariance function. Since this function is unevaluated within the Huygens-Fresnel approach, recourse is again necessary to first-order or saturation theories as discussed in Section II.B. It is interesting to note however that Eq. (105) is consistent with Eq. (8) of Ref. 19, which was obtained through the more complicated phase-screen approach.[13]

In the case of a small target spot (diameter$<(L/k)^{1/2}$) and unsaturated scintillations, Eq. (105) is a statement of point-source behavior. For a larger target spot, it appears that the covariance scale will be smeared by geometrical effects, and this can be directly deduced from the spatial-spectrum representation of Ref. 19, Eq. (9). (Note however that the quan-

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18. D.L.Fried, "Spectral and Angular Covariance of Scintillation for Propagation in a Randomly Inhomogeneous Medium",Applied Optics,10,721,Apr.1971.
19. S.F.Clifford,G.R.Ochs,and Ting-i Wang,"Optical Wind Sensing by Observing the Scintillations of a Random Scene",Applied Optics,14,2844,Dec. 1975.

tity q in that reference is necessarily real, contrary to the notation there. Also, q and χ in that reference may be identified with $|U|$ or $I^{1/2}$ so that the calculated covariance is closely related to but not identical to that for I.) It is known from other analyses[20] that the scintillation variance is reduced or "smoothed" by a featureless, extended incoherent source, specifically

$$\sigma_{I_N}^2 \sim [D/(L/k)]^{-7/3} \tag{106}$$

where D is the source diameter. This is also the "aperture smoothing factor" for an incoherent receiver in unsaturated scintillations.[20]

In the case of saturated scintillations ($\rho_0 < (L/k)^{1/2}$), the covariance function in Eq. 105 must be suitably modified. The behavior of the irradiance correlation function, in particular its scale size, is not well understood for this case of incoherent illumination and strong turbulence scattering.

II.E. Comparison of Alternative Approaches; Variance and Covariance Behavior vs. Parameter Realms

The basic method of analysis of this report is the extended Huygens-Fresnel principle, which is uniquely suited to expressing problems of arbitrary sources in terms of point-source quantities. It also has the virtue of inherently retaining the "saturation" or multiple scattering effect including the explicit influence of the quantity ρ_0 e.g for an extended, coherent source.

The most elegant alternative method is the phase-screen approach,[13,19] a first-order (single-scattering) analysis which also handles an arbitrary source (and receiver) geometry but with severe limitations on the strength of scattering or phase distortion for extended coherent sources. It appears that this approach can be made to handle saturation effects properly through modification of a log amplitude covariance function,[14-17] although general applicability e.g. to ρ_0-effects over a coherent source has not yet been clearly demonstrated.

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20. R. F. Lutomirski, et al, Degradation of Laser Systems by Atmospheric Turbulence, Rand Report R-1171-ARPA/RC, June 1973.

The two approaches are basically complementary when properly compared. The Huygens-Fresnel approach is much simpler in its handling of extended monochromatic sources (including the diffuse target case), and also simpler in its setting up of the extended, incoherent case. However, in general it leaves unevaluated the point-source log-amplitude covariance, which must then be obtained from the first-order theory and saturation arguments described above. Also, source structure may be advantageously represented in terms of its spatial spectrum, as shown in Ref. 19.

A very useful "hybrid" approach would be to utilize the saturation C_χ function in the Huygens-Fresnel results for various sources. Also, it is of interest to explore the behavior of that function for nonmonochromatic sources: how sensitive is the influence of ρ_0 to spectral width, i.e. is ρ_0 "washed out" as readily as interferometric phenomena such as speckles?

We are now in a position to summarize the variance and covariance behavior, for a variety of sources, in a particularly revealing manner. We will identify the fundamental independent variables and all possible permutations of their mutual inequalities (asymptotic parameter realms), and attempt to show the limiting value of the variances and covariance scales for each such realm, and for each source of interest. In order to do this we will utilize results from phase-screen or first-order theory and from Huygens-Fresnel analyses when respectively appropriate. This type of summary has the advantage of clear physical interpretation and lacks only the quantitative details of the transition between parameter realms. Also, in some cases it permits the deduction of e.g. covariance scales through dimensional analysis or the requirement of consistency with adjacent parameter realms.

We present these results in tabular form, with associated remarks below. In all cases, the right hand column represents the saturated or multiple scattering realm as opposed to the single-scattering realm in the left-hand column. Also, "near-field" and "far-field" refer to the position of the (point) on-axis receiver relative to the source size. The equalities shown omit numerical constants.

The symbols are defined as follows:

Path length:	L
Wavenumber:	k
Coherence radius:	ρ_o
Gaussian-beam transmitter radius:	α_o
Incoherent transmitter diameter:	b
Covariance scale (scintillation patch size) at receiver:	ρ_a
Beam size at receiver:	w
Log amplitude variance:	σ_χ^2
Normalized irradiance variance:	$\sigma_{I_N}^2$

Note: for log normal scintillations $\sigma_{I_N}^2 = e^{4\sigma_\chi^2} - 1$.

Table I:

Focused TEM_{00} beam. Realm (a) is the first-order point-source realm, while (b) is the "transmitter aperture smoothing" realm within the first-order theory.[24-27] It is interesting to note that the latter has the same predicted power-law dependence of the variance on the transmitter size as that for the incoherent source discussed in Section II.D. The reason for this is that in both cases, interference does not play a role: in the incoherent case, there is no interference, and in the coherent case, with $\rho_o > \alpha_o$, there is insignificant wavefront distortion. Realm (c) represents significant wavefront interference, a proliferation of diffraction scale patches,[26] and beamspread determined by diffraction from ρ_o. Realms (d,e) represent a saturated point source,[14-16] and realm (f) represents a large source with multiple scattering. In the latter case, which has not been experimentally verified, the covariance scale is deduced from adjacent realms (c,e) by the requirement of consistency at their respective transitions. In all cases involving beam breakup or saturation, the normalized variance is unity in agreement with a model of random-phase, identical-frequency summed oscillators.[8]

24. D.L.Fried and J.B.Seidman,"Laser Beam Scintillation in the Atmosphere", JOSA, 57, 181, February 1967.
25. A.Ishimaru, "Fluctuations of a Beam Wave Propagating Through a Locally Homogeneous Medium", Radio Science, 4, 295, April 1969.
26. J.R.Kerr and R.Eiss, "Transmitter-Size and Focus Effects on Scintillations", JOSA, 62, 682, May 1972.
27. J.R.Kerr and J.R.Dunphy, "Experimental Effects of Finite Transmitter Apertures on Scintillations", JOSA, 63, 1, January 1973.

TABLE I. Focused TEM_{00} Beam (α_o)

	$\sqrt{L/k} < \rho_o$	$\sqrt{L/k} > \rho_o$
$\alpha_o < \sqrt{L/k}, \rho_o$ (Far Field)	A $(\alpha_o < \sqrt{L/k} < \rho_o)$ $\rho_a = \sqrt{L/k}$ $w = \frac{L}{k\alpha_o}$ $\sigma_\chi^{\ 2} = C_n^{\ 2} k^{7/6} L^{11/6}$	D $(\alpha_o < \rho_o < \sqrt{L/k})$ $\rho_a = \rho_o$ $w = L/k\alpha_o$ $\sigma_{I_N}^{\ 2} = 1$
$\sqrt{L/k} < \alpha_o < \rho_o$ (Near Field)	B $\rho_a = \frac{L}{k\alpha_o}$ $w = \frac{L}{k\alpha_o}$ $\sigma\chi^2 \sim (\alpha_o)^{-7/3}$	
$\rho_o < \alpha_o < \sqrt{L/k}$ (Far Field)		E $\rho_a = \rho_o$ $w = \frac{L}{k\rho_o}$ $\sigma_{I_N}^{\ 2} = 1$
$\alpha_o > \sqrt{L/k}, \rho_o$ (Near Field)	C $(\sqrt{L/k} < \rho_o < \alpha_o)$ $\rho_a = L/k\alpha_o$ $w = L/k\rho_o$ $\sigma_{I_N}^{\ 2} = 1$	F $(\rho_o < \sqrt{L/k} < \alpha_o)$ $\rho_a = \frac{\rho_o \sqrt{L/k}}{\alpha_o}$ $w = \frac{L}{k\rho_o}$ $\sigma_{I_N}^{\ 2} = 1$

Table II:

Collimated TEM_{00} beam. Realms (a,d,e) are identical to those in Table I. Realms (b,c) are essentially plane-wave illumination within the first-order theory, while (f) represents plane-wave illumination with saturation.

Table III:

This is the case analyzed in Sections II.A,B, in which a focused TEM_{00} beam illuminates a diffuse target. The target is the "source" and the receiver is in the transmitter plane (Figure 1). "Near and far field" refer to the receiver-plane relationship to the illuminated spot on the target, which in some realms is in turn determined by the ρ_0 diffraction scale. Realms (a,b) are the first-order point source case with log normal scintillation statistics. In realms (d,e) the "atmospheric speckle" predominates, with joint gaussian field statistics, while in (c,f) the target speckle predominates, also with joint gaussian statistics. In realms (c-f) the model of independent additive oscillators applies, resulting in a normalized variance of unity.[8] In the transition region between (e,f), the field statistics are gaussian but not joint gaussian. In the transition region between (a,d), the normalized irradiance variance may exceed unity. The amplitude perturbation term (C_χ) is important (Section II.B) in realms (a,b). In fact, Figures 3a,c correspond to realms a (or b) and d, respectively.

Table IV:

Collimated TEM_{00} beam illuminating a diffuse target. This case is only of interest for $\alpha_0{}^2>L/k$, such that "collimation" has meaning (near field conditions). The basic considerations are the same as for Table III. (We also note here a further possibility, that of a direct monochromatic diffuse source (such as an extended laser transmitter with a diffusing screen in front of it.) In this case, Table I applies with the only exception that $w = \infty$ and, in realm (b), $\sigma_{I_N}{}^2 = 1$.)

Table V:

Incoherent (Nonmonochromatic) Source. In this case we assume that the spectral width of the source is not sufficient for "spectral diversity"

TABLE II. Collimated TEM_{00} Beam (α_o)

	$\sqrt{L/k} < \rho_o$	$\sqrt{L/k} > \rho_o$
$\alpha_o < \sqrt{L/k}\ ,\rho_o$ (Far Field)	A $(\alpha_o < \sqrt{L/k} < \rho_o)$ $\rho_\alpha = \sqrt{L/k}$ $w = \frac{L}{k\alpha_o}$ $\sigma_\chi^{\ 2} = C_n^{\ 2} k^{7/6} L^{11/6}$	D $(\alpha_o < \rho_o < \sqrt{L/k})$ $\rho_a = \rho_o$ $w = L/k\alpha_o$ $\sigma_{I_N}^{\ 2} = 1$
$\sqrt{L/k} < \alpha_o < \rho_o$ (Near Field)	B $\rho_a \approx \sqrt{L/k}$ $w = \alpha_o$ $\sigma_\chi^{\ 2} = C_n^{\ 2} k^{7/6} L^{11/6}$	
$\rho_o < \alpha_o < \sqrt{L/k}$ (Far Field)		E $\rho_a = \rho_o$ $w = \frac{L}{k\rho_o}$ $\sigma_{I_N}^{\ 2} = 1$
$\alpha_o \geq \sqrt{L/k}\ ,\rho_o$ (Near Field)	C $(\sqrt{L/k} < \rho_o < \alpha_o)$ $\rho_a = \sqrt{L/k}$ $w = \alpha_o + \frac{L}{k\rho_o}$ $\sigma_\chi^{\ 2} = C_n^{\ 2} k^{7/6} L^{11/6}$	F $(\rho_o < \sqrt{L/k} < \alpha_o)$ $\rho_a = \rho_o$ $w = \alpha_o + \frac{L}{k\rho_o}$ $\sigma_{I_N}^{\ 2} = 1$

TABLE III. Diffuse Target/Focused Beam (α_o)
(Note: $w = \infty$)

$\sqrt{L/k} < \rho_o$	$\sqrt{L/k} > \rho_o$
A ($\sqrt{L/k} < \rho_o < \alpha_o$ - Far Field) $\rho_a = \sqrt{L/k}$ $\sigma_\chi^{\ 2} = C_n^{\ 2} k^{7/6} L^{11/6}$	D ($\rho_o < \sqrt{L/k} < \alpha_o$ -Near Field) $\rho_a = \rho_o$ $\sigma_{I_N}^{\ 2} = 1$
B ($\sqrt{L/k} < \alpha_o < \rho_o$ -Far Field) $\rho_a = \sqrt{L/k}$ $\sigma_\chi^2 = C_n^{\ 2} k^{7/6} L^{11/6}$	
	E ($\rho_o < \alpha_o < \sqrt{L/k}$ -Near Field) $\rho_a = \rho_o$ $\sigma_{I_N}^{\ 2} = 1$
C ($\alpha_o < \sqrt{L/k} < \rho_o$ -Near Field) $\rho_a = \alpha_o$ $\sigma_{I_N}^{\ 2} = 1$	F ($\alpha_o < \rho_o < \sqrt{L/k}$ - Near Field) $\rho_a = \alpha_o$ $\sigma_{I_N}^{\ 2} = 1$

TABLE IV. Diffuse Target/Collimated Beam (α_o)
(Note; $w = \infty$)

$\sqrt{L/k}<\rho_o$	$\sqrt{L/k}>\rho_o$
A ~~($L/k<\rho_o < L/k\alpha_o$)~~	D ~~($\rho_o<\sqrt{L/k}< L/k\alpha_o$)~~
B ~~($\sqrt{L/k}< L/k\alpha_o<\rho_o$)~~	
	E ($\rho_o<L/k\alpha_o<\sqrt{L/k}$) $\rho_a = \rho_o$ $\sigma_{I_N}^2 = 1$
C ($L/k\alpha_o<\sqrt{L/k}<\rho_o$) $\rho_a = L/k\alpha_o$ $\sigma_{I_N}^2 = 1$	F ($L/k\alpha_o<\rho_o<\sqrt{L/k}$) $\rho_a \approx L/k\alpha_o$ $\sigma_{I_N}^2 = 1$

TABLE V. Incoherent Source (b)
(Note: $w = \infty$)

	$\sqrt{L/k}<\rho_o$	$\sqrt{L/k}>\rho_o$
$b<\sqrt{L/k},\rho_o$ (Far Field)	A ($b<\sqrt{L/k}<\rho_o$) $\rho_a = \sqrt{L/k}$ $\sigma_\chi^2 = C_n^2 k^{7/6} L^{11/6}$	D ($b<\rho_o<\sqrt{L/k}$) $\rho_a \overset{?}{=} \rho_o$ $\sigma_{I_N}^2 \overset{?}{=} 1$ (see discussion)
$\sqrt{L/k}<b<\rho_o$ (Near Field)	B $\rho_a = b$ $\sigma_\chi^2 \sim (b)^{-7/3}$	
$\rho_o<b<\sqrt{L/k}$ (Far Field)		E $\rho_a \overset{?}{=} \rho_o$ $\sigma_{I_N}^2 \overset{?}{=} 1$ (see discussion)
$b>\sqrt{L/k},\rho_o$ (Near Field)	C ($\sqrt{L/k}<\rho_o<b$) $\rho_a = b$ $\sigma_\chi^2 \sim (b)^{-7/3}$	F ($\rho_o<\sqrt{L/k}<b$) (see discussion)

scintillations, i.e. smoothing owing to broadband effects.[18] Unfortunately, the incoherent case is not yet perfectly understood. Realm (a) is a point source in the nonsaturated case. Realms (b,c) correspond to an extended source also in the single-scattering case, for which the analysis of Ref. 20 predicts a -7/3 power-law behavior of the variance, and with a covariance scale that is related to the source size through geometrical effects (Ref. 19 and Section II.D); since the source is incoherent, the condition $\rho_o < b$ in going from realm (b) to (c) has no effect. Realms (d,e) are for the saturated point-source, and it is expected that as the spectral width increases over that for the monochromatic case, the "atmospheric speckles" (ρ_o) are washed out, although the degree of spectral spread required to cause this is not understood. The variance will decrease from unity when the detector has insufficient bandwidth to detect the "beats" between the individual oscillators comprising the nonmonochromatic source. Further collapse is expected for the multiple-scattering, near-field case of realm (f). As mentioned previously, it would be desirable to explore (d-f) further using the "saturation covariance" work of Refs. 14-17.

II.F. Further Work and Applications

The application of the efforts described in this report is to the performance of coherent optical adaptive (COAT) systems. The desirable analytical and experimentally verified results include the strength, spatial scale, and spatial and temporal spectrum of scintillations and their statistics, for a variety of sources including single- and multi-mode illuminated diffuse and structured targets. The coherent or complex amplitude correlation (mutual coherence function) of the receiver field is equally pertinent.

Finally, the turbulence effects should be applied to control system models of actual adaptive systems which are generally phase correcting systems. We have given this preliminary attention with the assistance of systems engineers,[28] including the dynamics of target lock-on and the resultant evolution in spot and speckle conditions.

Significant progress has been made on each of these problems, as described above.

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28. We acknowledge with appreciation the collaboration of James E. Pearson and others at the Hughes Research Laboratories, Malibu, California.

We detail below specific topics requiring further analytical work:

Completion of Fundamental Understandings

A. Complete the coherent-source, diffuse target covariance analysis of Section II.B., utilizing the full point-source mutual coherence function including C_χ terms. This primarily reduces to thorough numerical analysis of the integral results; also, the "saturation" form of C_χ (Refs. 14-17) should be incorporated. The results should be extended to the time-lapsed covariance, temporal and spatial spectra, and probability distribution of irradiance.

B. Complete a similar analysis with glints and other target structure, i.e. through second-irradiance moments and spectra (Section II.C.), and including the complex amplitude (mutual coherence function).

C. Incorporate incoherent receiver-aperture smoothing.

D. Incorporate multimode sources, and the extreme of incoherent sources (Section II.D.) including the saturation form of C_χ. In particular, the incoherence or spectral width required to wash out "atmospheric speckles" (ρ_0 scintillation scale) should be understood.

Applications

A. Characterize target models at wavelengths of interest and incorporate them into the analysis.

B. Analyze the effects of moving targets.

C. Analyze the influence of all turbulence effects on the operation of real adaptive systems, including lock-on dynamics.

Secondary Aspects

A. Influence of $\langle\chi\phi\rangle$ or the phase-amplitude correlation in the point-source mutual coherence function.

B. Effects of inner scale of turbulence.

III. Experimental Effort

The experimental effort is proceeding in three stages. The first involves preliminary experiments over a short (100m) path with a Helium-Neon laser operating at 6328 Å, with a quasi-diffuse (Scotchlight) target.

This project was designed to permit shakedown of the experimental techniques and system electronics and verification of basic theoretical predictions at an early date, and has been completed. The second stage involves operation over a 1 km uniform path with an Argon laser operating at 4880Å using a similar target. The required installations are nearly complete at our instrumented field site. The third stage involves the use of a TEA laser (3.5μ and 10.6μ) which is currently being fabricated and which will also be utilized at the field installation. Characterization of target models will also be conducted in the laboratory at wavelengths of interest, and target structure will be added at the field site.

A basic problem in these experiments is the energy available off of a diffuse or quasi-diffuse target at a distance. At visible wavelengths, the directional properties of Scotchlight permit an adequate signal-to-noise ratio while preserving the pertinent properties of a "diffuse" target; that is, the speckles in the absence of turbulence have identical properties to those observed for a truly diffuse source. At the infrared wavelengths, the high instantaneous peak power of the TEA laser is required.

The project goal is the observation and measurement of a number of statistical properties as described in the analytical discussion of Section II. The basic system is illustrated in Figure 5. The coherent laser illumination is modulated, focused, spatially filtered, and expanded to impinge on the remote target surface. The output beam may be focused, collimated, or operated in an intermediate condition. The return energy is detected by two separable receivers located adjacent to the transmitter, demodulated, filtered, and recorded on analog tape for digital processing. Simultaneously, the correlation of the two irradiance signals is computed electronically with variable time averaging and displayed. The measurements are repeated at various receiver separations to yield a covariance curve, and the turbulence strength is measured using a microthermal probe system. The output from final processing includes

Turbulence level (C_n^2 and ρ_o)
Spatial covariance function
Probability distribution of irradiance

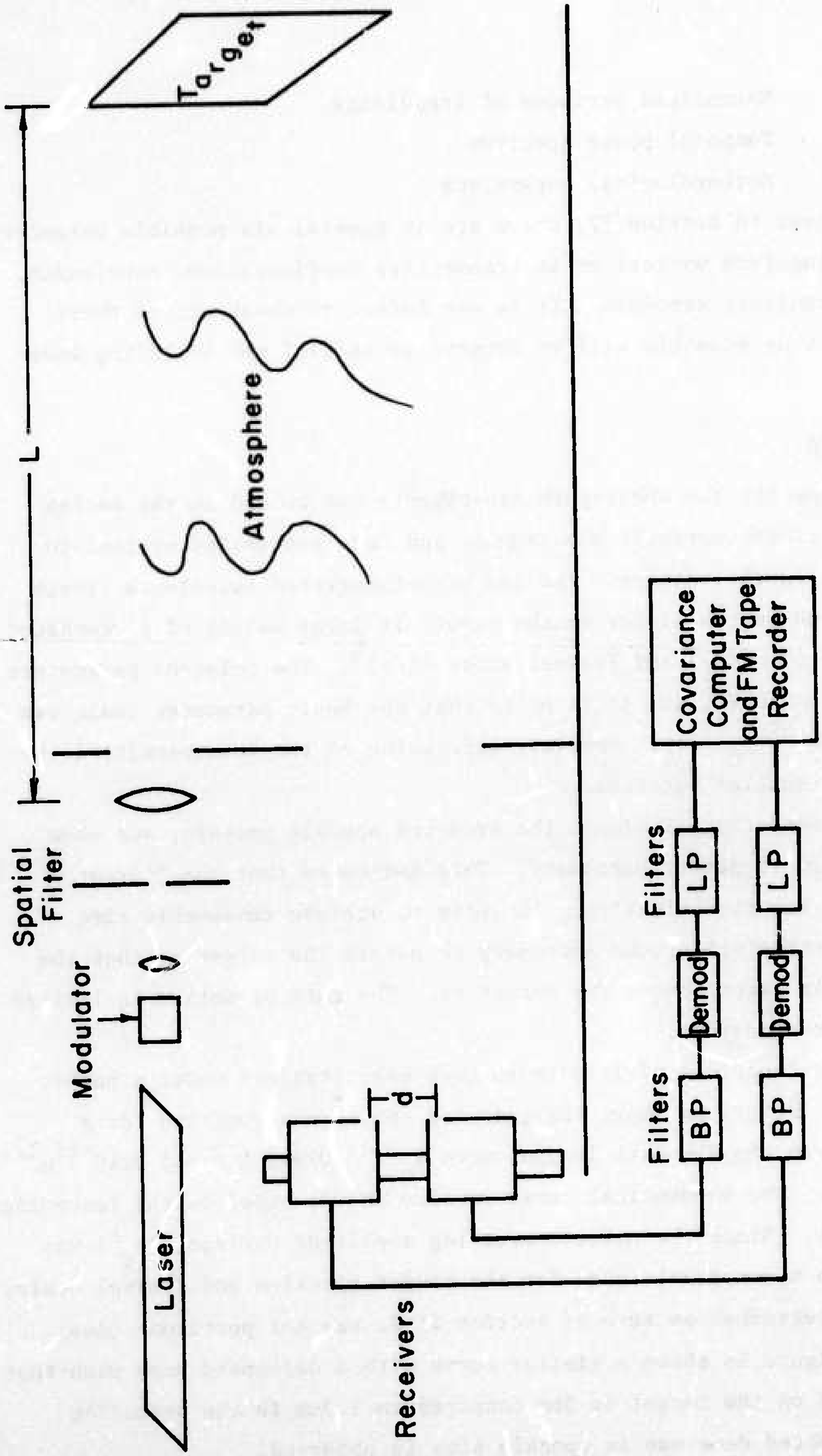

Figure 5. Experimental System

Normalized variance of irradiance

Temporal power spectrum

Meteorological parameters

As discussed in Section II, there are in general six possible parameter realms resulting from variations in transmitter configuration, wavelength, range, and turbulence strength. It is our intent to cover all of these realms; as many as possible will be covered at each of the operating wavelengths.

Initial Results

The purpose for the short-path experiments was to aid in the design and shakedown of the optical, electronic, and data processing systems to be used in the further stages. The low path-integrated turbulence levels over such a path in the winter months result in large values of ρ_0 compared to transmitter size (α_0) and Fresnel scale $(L/k)^{\frac{1}{2}}$. The relevant parameters are listed in Table VI, and it is noted that the basic parameter realm was limited to $(L/k)^{\frac{1}{2}} \lesssim \alpha_0 < \rho_0$. However, defocusing of the beam permitted the measurement of smaller speckles.

Visual observations indicate the expected speckle pattern, and show that the pattern is nearly sationary. This indicates that the "target speckles" (α_0) are predominating. In order to achieve reasonable time averaging, it therefore proved necessary to rotate the target so that the speckle field is swept across the detectors. The rate of motion is limited by the receiver bandwidth.

Covariance functions of irradiance have been obtained under a number of conditions. Figure 6a shows the measured covariance function for a focused beam over the 85m path length, with α_0 = 0.013m, $C_n^2 = 5.0\times10^{-15} m^{2/3}$ and ρ_0 = 0.15m. The theoretical curve is also shown, based on the reasonings of Section II.A. Since the point-source log amplitude variance (σ_χ^2) was quite low, with a comparable size for the target speckles and Fresnel scale, the amplitude perturbation term of Section II.B. was not pertinent (see Figure 3). Figure 6b shows a similar curve with a defocused beam such that the spot radius on the target is 5mm compared to 1.3mm in the preceding case. The expected decrease in speckle size is observed.

TABLE VI. Experimental Parameters

	Stage 1	Stage 2
Wavelength	$.6328\mu$	0.488μ
Range	85m	100-1000m
Transmitter Beam Radius α_o	.013m	.005-.02m
$\sqrt{L/k}$	.0029m	.0028-.0088m
$\sqrt{\lambda L}$	.0073m	.007-.022m
Range of ρ_o values	$0.1\sim1.0$ meter	1mm-10cm (including summer conditions)
Laser Power	$\sim$50 milliwatts	$\sim$1.0 watt
Modulation Frequency	9 kHz	100 kHz
Signal Bandwidth	1 kHz	$\geq$ 1 kHz
Receiver Size	2 mm	2 mm
SNR (Scotchlight)	$>10^4$	$>10^4$
SNR (True Diffuse)		10^3 (Night conditions, 100m path)

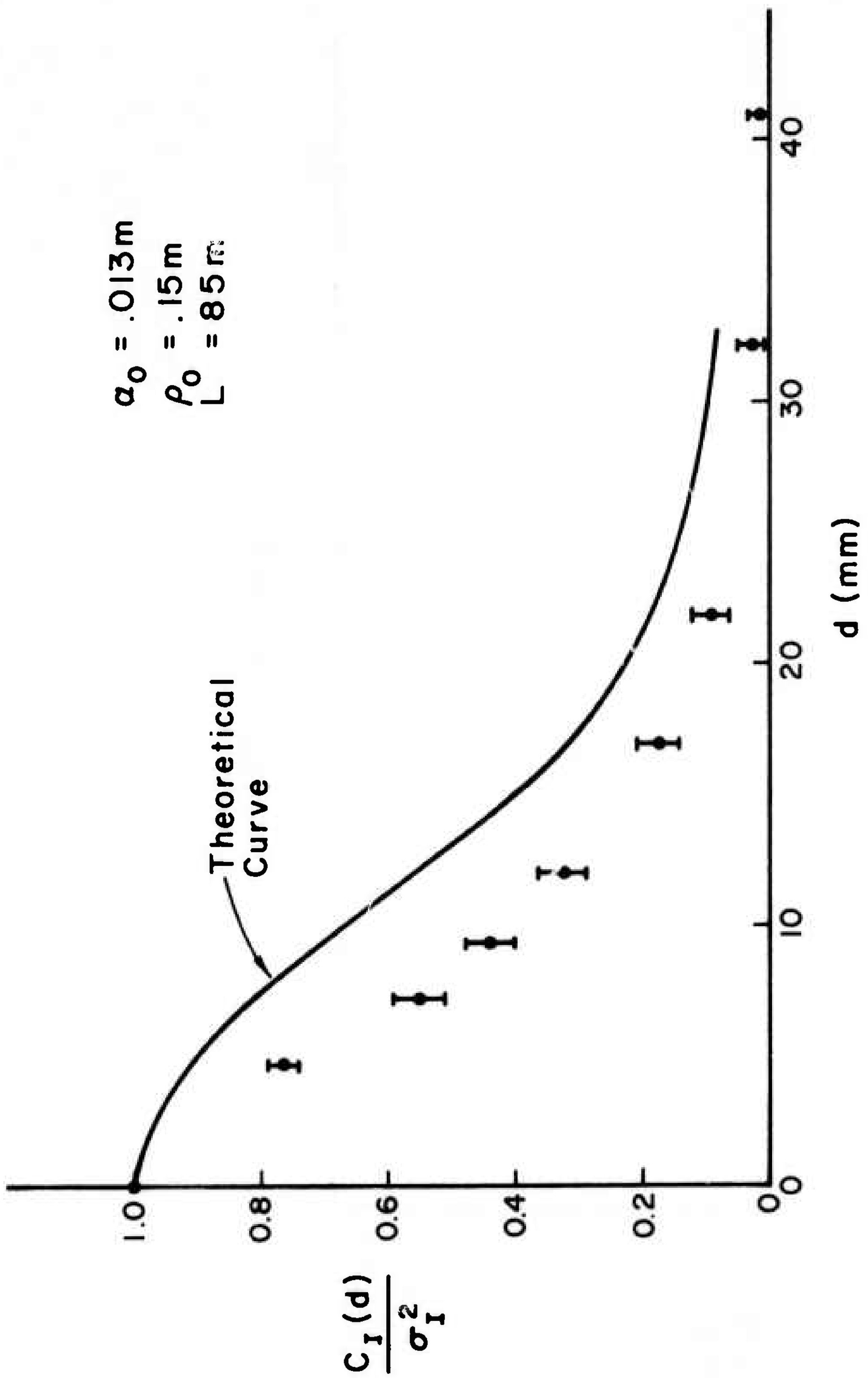

Figure 6a. Measured covariance of irradiance for focused beam and parameters shown.

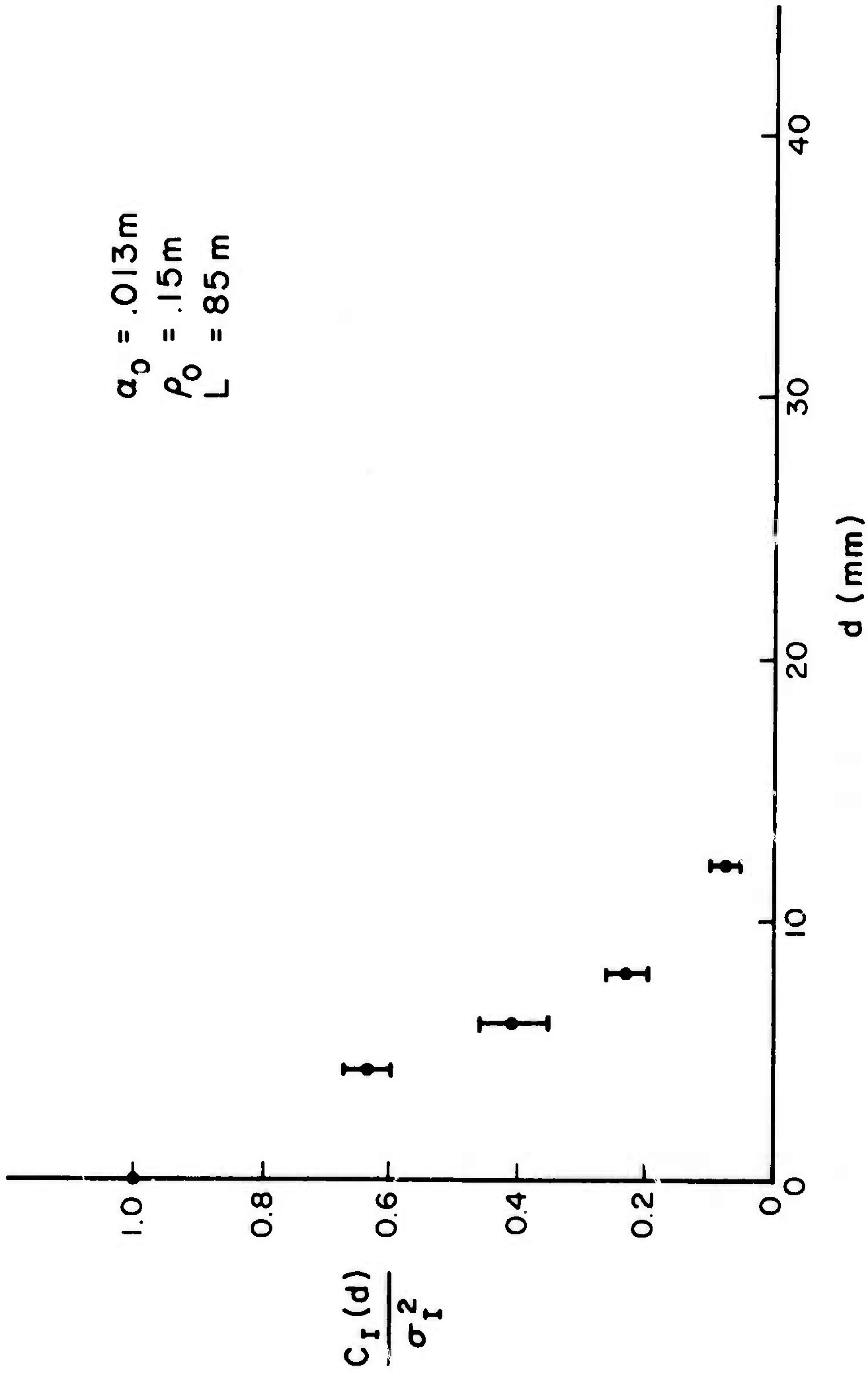

Figure 6b. Measured covariance of irradiance for defocused beam and parameters shown.

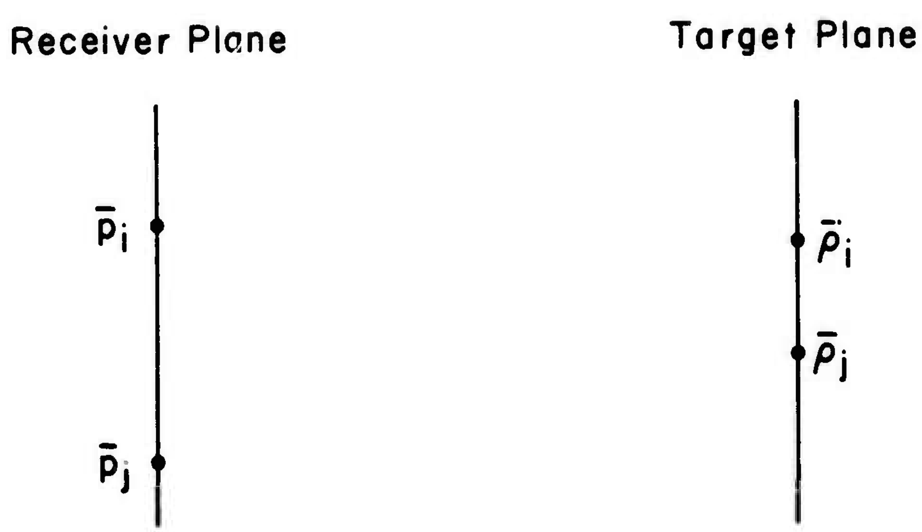

Figure 7. Variables for mutual coherence function.

Long-time-averaged ($\sim$ 6 minutes) measurements of the normalized variable of irradiance are summarized below, for the focused case:

$C_n^2 (m^{-2/3})$	$\sigma^2/\bar{I}^2$
8.2×10^{-16}	1.12
8.9×10^{-16}	0.85
4.5×10^{-15}	1.16

The theoretical value is of course unity.

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Appendix A. Generalized Mutual Coherence Functions

The point-source mutual coherence functions are crucial to the utilization of the extended Huygens-Fresnel approach. In this Appendix we review the generalized, high-order mutual coherence function. The derivation will not be given but is based upon considerations suggested by Yura.[2,21]

The general n^{th} order mutual coherence function is

$$H(\bar{\rho}_1, \bar{\rho}_2 \ldots \bar{\rho}_{2n}; \bar{p}_1, \bar{p}_2 \ldots \bar{p}_{2n})$$

$$= \left\langle \exp \left\{ \psi(\bar{\rho}_1, \bar{p}_1) + \psi^*(\bar{\rho}_2, \bar{p}_2) + \ldots + \psi(\bar{\rho}_{2n-1}, \bar{p}_{2n-1}) + \psi^*(\rho_{2n}, p_{2n}) \right\} \right\rangle \quad (A1)$$

where $\psi = \chi + i\phi$.

We now make the following assumptions:

1. χ has a joint normal distribution, with $\langle\chi\rangle = -\sigma_\chi^2$.
2. ϕ has a joint normal distribution, with $\langle\phi\rangle = 0$.
3. χ and ϕ are independent.

There are limitations on the validity of these assumptions. It is known that χ is not gaussian in the case of saturated scintillations, and as pointed out in the preceding report on this program, it is in general not jointly normal.[3] Also, $\langle\chi\phi\rangle$ is in general nonzero and can be calculated from the first-order theory.

Assumption #1 can be replaced with the assumption of a gaussian refractive index field, but this is a very poor representation of the actual nature of that field.[22] Alternatively,[2] we may simply say that the results to be given here are correct to "second order in the refractive index variations", which is at best a heuristic approximation in conditions of saturated scintillations. However, we will not dwell further here on the degree of approximation involved in the assumptions, which comprises a fruitful area for further work. It is sufficient to point out that very useful results are obtained.

The primary result is as follows:

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21. H. T. Yura, private communication.
22. J.R. Kerr, et al, "Propagation of Multiwavelength Laser Radiation through Atmospheric Turbulence", RADC-TR-74-320, November 1974, (A003340).

$$H_n(\bar{\rho},\bar{p}) = \exp\left\{-\frac{1}{2}\sum_{j=i+1}^{2n}\sum_{i=1}^{2n-1}(-1)^{i+j+1}D_{\psi_{ij}} + 2\sum_{j=1}^{M}\sum_{i=1}^{2n-2}C_{\chi_{2j+i,i}}\right\} \tag{A2}$$

where M is the number of j satisfying 2j+i=2n (for i even) and 2j+i=2n-1 (for i odd). The wave structure function D_ψ and log amplitude covariance function C_χ comprise a generalization of those normally seen in the literature.[23] Referring to Figure 7, they refer to complex phase or log amplitude quantities as a function of $(\bar{p}_j-\bar{p}_i)$ in the observation plane, for fields emanating from $\bar{\rho}_j$ and $\bar{\rho}_i$. Specifically,

$$\frac{1}{2}D_{\psi_{ij}} = \frac{1}{2}D\left[(\bar{\rho}_j-\bar{\rho}_i),\ (\bar{p}_j-\bar{p}_i)\right]$$

$$= \exp\left\{-\frac{1}{\rho_o^{5/3}}\left[\frac{|\bar{p}_j-\bar{p}_i|^{8/3}-|\bar{\rho}_j-\bar{\rho}_i|^{8/3}}{|(\bar{p}_j-\bar{p}_i)-(\bar{\rho}_j-\bar{\rho}_i)|}\right]\right\} \tag{A3}$$

$$\underset{\text{(first-order)}}{C\chi((\bar{\rho}_j-\bar{\rho}_i),(\bar{p}_j-\bar{p}_i))} = 0.132\pi^2 k^2\int_o^L ds C_n^2(s)$$

$$\cdot\int_o^\infty du\, u^{-8/3}\sin^2\left[\frac{u^2 s(L-s)}{2kL}\right] J_o\left[u\left|(\bar{p}_j-\bar{p}_i)\frac{s}{L} + (\bar{\rho}_j-\bar{\rho}_i)(1-\frac{s}{L})\right|\right] \tag{A4}$$

It may be noted that the functions satisfy $(\bar{p},\bar{\rho})$ reciprocity.

Special Cases

(a) n = 1

$$H(\bar{\rho}_1,\bar{\rho}_2;\bar{p}_1,\bar{p}_2) = e^{-\frac{1}{2}D_{\psi_{12}}} \tag{A5}$$

(b) n = 2

$$H(\bar{\rho}_1,\bar{\rho}_2,\bar{\rho}_3,\bar{\rho}_4;\bar{p}_1,\bar{p}_2,\bar{p}_3,\bar{p}_4) = e^{-\frac{1}{2}(D_{12}-D_{13}+D_{14}+D_{23}-D_{24}+D_{34})+2C_{\chi_{13}}+2C_{\chi_{24}}} \tag{A6}$$

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23. D. L. Fried, "Atmospheric Modulation Noise in an Optical Heterodyne Receiver", IEEE Trans. on Quantum Electronics, QE-3, 213, June 1967.

This is a generalization (double-arguments) of the result of Ref. 23

(c) n = 2: $\bar{p}_1=\bar{p}_2, \bar{p}_3=\bar{p}_4$ (calculation of $\langle I_1 I_2 \rangle$)
See Eqs. (18,20) in Section II.

(d) n = 2; $\bar{p}_1=\bar{p}_2, \bar{p}_3=\bar{p}_4; \bar{\rho}_1=\bar{\rho}_2=\bar{\rho}_3=\bar{\rho}_4$ (calculation of $\langle I_1 I_2 \rangle$ for point source)

$$H = e^{4C_\chi}(\bar{p}_3-\bar{p}_1) \qquad \text{(A7)}$$

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